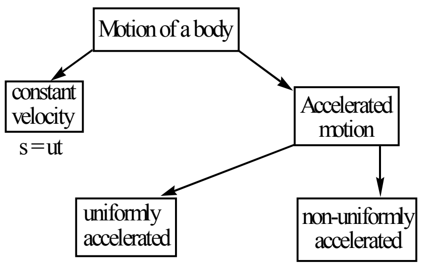
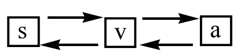


$v = u + at$

$v = \frac{ds}{dt}; a = \frac{dv}{dt} = v\frac{dv}{ds}$

$s = \left(\frac{u+v}{2}\right)t$



$v^2 - u^2 = 2as$

$\int ds = \int v dt$

$S = ut + \frac{1}{2}at^2$

$\int dv = \int a\, dt$

$S_n = u + a\left(n - \frac{1}{2}\right)$

$\int a ds = \int v dv$

SOLVED PROBLEMS BASED ON KINEMATICAL EQUATIONS

Problem : 1

The displacement x of a particle at the instant when its velocity v is given by v= $\sqrt{3x+16}$. Find its acceleration and intial velocity

Sol. $v = \sqrt{3x+16}$ or $v^2 = 3x + 16$ or $v^2 - 16 = 3x$ Comparing with $v^2 - u^2 = 2aS$, we get , u = 4 units,2a= 3 or a = 1.5 units

Problem : 2

If $S_n = 2 + 0.4n$ ***find intial velocity and acceleration***

Sol. $S_n = 2 + 0.4n$

$\Rightarrow S_n = 2 + 0.4n - \frac{0.4}{2} + \frac{0.4}{2}$

$\Rightarrow S_n = 2.2 + 0.4\left(n - \frac{1}{2}\right)$

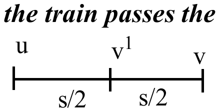
Compairing it with $S_n = u + a\left(n - \frac{1}{2}\right)$

U = 2.2 units a = 0.4 units

Problem : 3

The two ends of a train moving with uniform acceleration pass a certain point with velocity u and v. Find the velocity with which the middle point of the train passes the same point.

Sol.



$v^{12} - u^2 = 2a\ s/2$

$v^2 - v^{12} = 2as/2$

$\Rightarrow v^{12} - u^2 = v^2 - v^{12}$

$2v^{12} = v^2 + v^2$

$v^1 = \sqrt{\frac{u^2+v^2}{2}}$

Problem : 4

The velocity of a body moving with uniform acceleration of 0.3m/s² is changed to 3m/s in certain time. If the average velocity in the same time is 30m/s then find the distance travelled by it in that time.

Sol. v –u = at

$3 = 0.3t \Rightarrow t = 10s$

$S = V_{average}{}^{x\ time}$

$= 30 \times 10 = \boxed{300\,m}$

Problem : 5

A body starts from rest and moves with uniform acceleration of 5 ms^{-2} for 8 seconds. From that time the acceleration ceases. Find the distance covered in 12s starting from rest.

Sol. The velocity after 8 seconds v = 0 + 5× 8 = 40 m/s

Distance covered in 8 seconds

$$s_0 = 0 + \frac{1}{2} \times 5 \times 64 = 160\ m$$

After 8s the body moves with uniform velocity and distance covered in 4s with uniform velocity

v=vt= 40 × 4 = 160 m

The distance covered in 12 s =160 +160 = 320 m.

Problem : 6

A scooter can produce a maximum acceleration of 5m s^{-2}. Its brakes can produce a maximum retardation of 10 ms^{-2}. The minimum time in which it can cover a distance of 1.5 km is ?

Sol. If v is the maximum velocity attained, then

$v^2 - O^2 = 2 \times 5 \times S_1$. Also , $O^2 - v^2 = 2 \times 10 \times S_2$

$$S_1 = \frac{v^2}{10}, S_2 = \frac{v^2}{20}$$

$$S = S_1 + S_2 \Rightarrow 1500 = \frac{v^2}{10} + \frac{v^2}{20} = \frac{3v^2}{20} \text{ or}$$

$$v^2 = \frac{1500 \times 20}{3} = 10000 \quad \text{or } v = 100 \text{ m s}^{-1}$$

Problem : 7

The speed of a train is reduced from 60 km/h, to 15 km/h, whilst it travels a distance of 450 m. If the retrardation is uniform , find how much further it will travel before coming to rest ?

Sol. Here $u = 60 \times \frac{5}{18} = \frac{50}{3} m/s$

$$v = 15 \times \frac{5}{18} = \frac{25}{6} m/s$$

Using $v^2 = u^2 + 2as$, we get

$$\left(\frac{50}{3}\right)^2 = \left(\frac{25}{6}\right)^6 + 2 \times 9 \times 450$$

or $a = -\frac{125}{36 \times 12} m/s^2$

If s^1 is the further distance travelled before coming to rest, then

$$s^1 = \frac{v^2}{2a} = \frac{25}{6} \times \frac{25 \times 36 \times 12}{6 \times 2 \times 125} = 30m$$

Problem : 8

A particle is at x = + 5 m at t = 0, x = – 7 m at t = 6 s and x = + 2 m at t = 10s. Find the average velocity of the particle during the intervals
(a) t = 0 to t = 6s (b) t = 6s to t = 10s,
(c) t = 0 to t = 10 s.

From the definition of average velocity

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

(a) The average velocity between the times t = 0 to t = 6s

$x_1 = +5m, t_1 = 0, x_2 = -7\ m\ t_2 = 6s$

Hence $v_1 = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-7-5}{6-0} = -2ms^{-1}$

(b) The average velocity between the times t_2 = 6s to t_3 = 10 s is

$$v_2 = \frac{x_3 - x_2}{t_3 - t_2} = \frac{2-(-7)}{10-6} = \frac{9}{4} = 2.25ms^{-1}$$

(c) The average velocity between times t_1= 0 to t_3=10 s is

$$v_3 = \frac{x_3 - x_1}{t_3 - t_1} = \frac{2-5}{10-0} = -0.3ms^{-1}$$

Problem : 9

Velocity and acceleration of a particle at time t = 0 are $\vec{u} = (2\hat{i} + 3\hat{j})\, m/s$ and $\vec{a} = (4\hat{i} + 2\hat{j})\, m/s^2$ respectively. Find the velocity and displacement of particle at t = 2s.

Sol. Here, acceleration

$\vec{a} = (4\hat{i} + 2\hat{j})$ m/s^2 is constant. So, we can apply

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

substituting the proper values, we get

$$\vec{v} = (2\hat{i} + 3\hat{j}) + (2)(4\hat{i} + 2\hat{j}) = (10\hat{i} + 7\hat{j}) \text{ m/s}$$

and $\vec{s} = (2)(2\hat{i} + 3\hat{j}) + \frac{1}{2}(2)^2(4\hat{i} + 2\hat{j}) = (12\hat{i} + 10\hat{j})$ m

Therefore, velocity and displacement of particle at t = 2s are $(10\hat{i} + 7\hat{j})$ m/s and $(12\hat{i} + 10\hat{j})$ m respectively.

Problem : 10

A rifle bullet loses 1/20th of its velocity in passing through a plank. What will be the least number of such planks required to just stop the bullet ?

Sol. $\left(\frac{19v}{20}\right)^2 - v^2 = 2ax \quad 0^2 - v^2 = 2anx$

Dividing , $n = \frac{-v^2}{\left(\frac{19}{20}v\right)^2 - v^2} = \frac{1}{1 - \left(\frac{19}{20}\right)^2}$

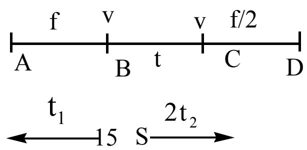
$$=\frac{20\times20}{(20+19)(20-19)}=\frac{400}{39}=10.3$$

$\Rightarrow$ 11 planks so, the bullet shall stop in 11 th plank.

Problem : 11

A car starting from rest, accelerates at the rate of f through a distance S, then continues at constant speed for time t and declerates at the rate $v=ft_1$ to come to rest. If the total distance travelled is 15 S then S =

Sol.



AB= $S=\frac{1}{2}f\,t_1^2$ -------- (1) $v=ft_1$

BC = $(ft_1)t$

CD = $\frac{u^2}{2a}=\frac{(ft_1)^2}{2(f/2)}$

$S_n=2+0.4m$

$S+f t_1 t+25=15\,S$

$ft_1t=12\,S$ -------- (2)

Dividing (1) by (2)

$t_1=\frac{t}{6}$

$S=\frac{1}{2}f\,\frac{t^2}{36}$ $\qquad \boxed{\therefore S=\frac{ft^2}{72}}$

Problem : 12

A body covers 100cm in first 2seconds and 128cm in the next two seconds moving with constant acceleration. Find the velocity of the body at the end of 8sec?

Sol. $100=2u+\frac{1}{2}a.4$ -------(1)

$228=6u+\frac{1}{2}a.36\rightarrow(2)$

$(1)\times3-(2)$ gives

$72=-12a$

$a=-6\ cm/s^2$

$100=2u-\frac{1}{2}\times6\times4$

$2u=112$ $\qquad$ $u=56cm/s$

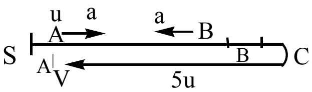
$V=u+at=56-6\times8$

$\therefore \boxed{V=8\,cm/s}$

Problem : 13

A body starts with initial velocity u and moves with uniform acceleration.when the velocity has increased to 5u, the acceleration is reversed in direction, the magnitude remaining constant. Find its velocity when it returns to the starting point?

Sol. AB = S



For AB $25u^2-u^2=2as$

$V^2-25u^2=2(-a)\times(-s)$

$\Rightarrow V^2-25u^2=24u^2$

$V^2=49u^2$ $\qquad$ $V=\pm7u$

$\therefore V$ is opposite to u $\boxed{V=-7u}$

Problem : 14

A train starting from rest travels the first part of its journey with constant acceleration a, second part with constant velocity v and third part with constant retardation a , being brought to rest. The average speed for the whole journey is $\frac{7v}{8}$. For what fraction of the total time, the train travels with constant velocity ?

Sol. $\frac{7v}{8}=\frac{\frac{1}{2}vt+vt^1+\frac{1}{2}vt}{t+t^1+t}$ or $\frac{7}{8}=\frac{t+t^1}{2t+t^1}$

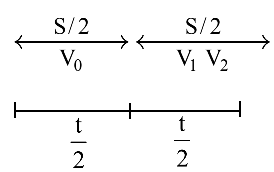
or $6t=t^1$

Now $\frac{t}{2t+t^1}=\frac{6t}{8t}=\frac{3}{4}$

Problem : 15

A particle traversed half of the distance with a veloicty of V_0. The remaining parts of the distance was covered with velocity V_1 for half of the time and with V_2 for other half of the time. Find the mean velocity of the particle averaged and the whole time of motion

Sol :



Average velocity for the second half distance =

$$\frac{v_1\frac{t}{2}+v_2\frac{t}{2}}{\frac{t}{2}+\frac{t}{2}}=\frac{v_1+v_2}{2}$$

Average velocity for the first half distance = $= v_0$

($\therefore$ it is constant)

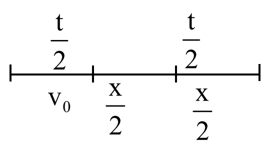
Average velocity for total path

$$= \frac{2v_0 \frac{(v_1+t_2)}{2}}{v_0+\frac{v_1+v_2}{2}} = \boxed{\frac{2v_0(v_1+v)}{v_1+v_2+2v_0}}$$

Problem : 16

A particle traversed along a straight line for first half time with velocity V_0. For the remaining part, half of the distance is traversed with velocity V_1 and other half distance with veloctity V_2. Find the mean velocity of the pariticle for the total journey.

Sol.



For the first half time average velocity = V_0

For the second half time average velocity = $\frac{2v_1v_2}{v_1+v_2}$

Average velocity for total journey $= \frac{v_0+\frac{2v_1v_2}{v_1+v_2}}{2}$

$$\boxed{\text{average velocity} = \frac{v_0(v_1+v_2)+2v_1v_2}{2(v_1+v_2)}}$$

Problem :17

A car is moving with a velocity of 20 m/s. The driver sees a stationary truck ahead at a distance of 100 m. After some reaction time Δt the brakes are applied producing a retardation of 4 m/s². What is the maximum reaction time to avoid collision ?

Sol. The car before coming to rest

$v^2 = u^2 + 2as$ covers distance s

$\therefore 0 = 20^2 - 2\times 4s$

$\therefore s = \frac{400}{8} = 50\ m$

The car covers 50 m

To avoid the clash, the remaining distance 100 - 50 = 50 m must be covered by the car with uniform velocity 20 m/s during the reaction time Δ t

$\therefore \frac{50}{\Delta t} = 20$ $\qquad \therefore \Delta t = \frac{50}{20} = 2.5\ s$

$\therefore$ The maximum reaction time $\Delta t = 2.5$ s

Problem : 18

A driver can stop his car from the red signal at a distance of 20m when he is driving at 36 kmph and 41.25m when he is driving at 54kmph. Find his reaction time.

$s = ut + u^2/2a$

$\left[20 = 10t + \frac{100}{2a}\right] \times 2.25$

$41.25 = 15t + \frac{225}{2a}$

3.75 = 7.5t

$\boxed{t = 0.5s}$

Problem : 19

A car starts from rest and moves with uniform acceleration 'a'. At the same instant from the same point a bike crosses with a uniform velocity 'u'. When and where will they meet ? what is the velocity of car with respect to the bike at the time of meeting?

Sol. $S_r = u_r t + \frac{1}{2}a_r t^2$

0= ut $- \frac{1}{2}at^2$

$\boxed{t = \frac{2u}{a}}$

$S_{bike} = u.t = u.\frac{2u}{a} = \frac{2u^2}{a}$ $\boxed{S_{bike} = \frac{2u^2}{a}}$

V_{car} =at = 2u

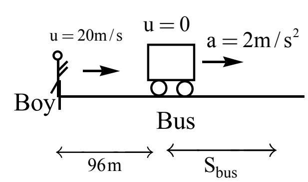
V_{car} w.r. t. bike at the time of meeting = 2u –u

$\boxed{V_{cb} = u}$

Problem : 20

A bus starts moving with acceleration 2m/s². A boy 96m behind the bus simultaneously starts running with a constant velocity of 20m/s. After what time he will be able catch the bus?

Sol.



$S_r = u_r t + \frac{1}{2}a_r t^2$

$96 = 20t - \frac{1}{2}2t^2$

$t^2 - 20t + 96 = 0$

on solving $\boxed{t = 8S}$ and $\boxed{t = 12S}$

He can catch the bus at two instants 8s and 12s. After 12 seconds the bus will always be ahead of the boy

Problem : 21

Two bodies start moving in the same straight line at the same instant of time from the same origin. The first body moves with a constant velocity of 40 m/s, and the second starts from rest with a constant acceleration of $4\ m/s^2$. Find the time that elapses before the second. catches the first body. Find also the greatest distance between then prior to it and the time at which this occurs.

Sol. When the second body catches the first , the distance travelled by each is the same.

$$\therefore 40t = \frac{1}{2}(4)t^2 \quad \text{or } t = 20\ \text{S}$$

Now , the distance s between the two bodies at any time t is $s = ut - \frac{1}{2}at^2$

For s to be maximum, $\frac{ds}{dt} = 0 \quad or \quad u - at = 0$

or $t = \frac{u}{a} = \frac{40}{u} = 10s$

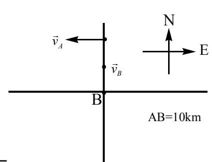
Maximum Distance

$$= 40\times 10 - \frac{1}{2}\times 4\times (10)^2 = 400 - 200 = 200m$$

Problem : 22

Two ships A and B are 10 km a part on a line running south to north . Ship A farthar north is streaming west at 20 km/hr and ship B is streaming north at 20 km/hr . What is their distance of closest approach and how long do they take to reach it ?

Sol. Ships A and B are moving with same speed 20 km/hr in the directions shown in figure . It is a two dimensinal , two body problem with zero acceleration



Let us find $\vec{v}_{BA}$

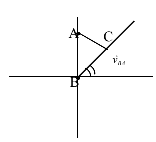
$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

here , $|\vec{v}_{BA}| = \sqrt{(20)^2 + (20)^2} = 20\sqrt{2}km/hr$

i.e $\vec{v}_{BA}$ is $20\sqrt{2}$ km/hr at an angle of 45^0 from east towards north . Thus , the given problem can be simplified as A is rest and B is moving with $\vec{v}_{BA}$ in the direction shown in

Therefore , the mininum distance between the two is

$$S_{\min} = AC = AB\sin 45^0 = 5\sqrt{2}km$$



and the desired time is

$$t = \frac{BC}{|\vec{v}_{BA}|} = \frac{5\sqrt{2}}{20\sqrt{2}} \quad \left(BC = AC = 5\sqrt{2}\right)$$

$\frac{1}{4}hr = 15$ minutes

Problem : 23

Two trains one travelling at 54 kmph and the other at 72 kmph are headed towards one another along a straight track. When they are $\frac{1}{2}$ km apart, both drivers simultaneously see the other train and apply their brakes. If each train is decelerated at the rate of $1\ ms^{-2}$, will there be collision ?

Sol. Distance travelled by the first train before coming to rest

$$s_1 = \frac{u^2}{2a} = \left(72\times\frac{5}{18}\right)^2 / 2\times 1 = \frac{400}{2} = 200\ m$$

$$= \frac{225}{2} = 122.5\ m$$

Distance travelled by the second train before coming to rest

$$s_2 = \left(72\times\frac{5}{18}\right)^2 / 2\times 1 = \frac{400}{2} = 200\ m$$

Total distance travelled by the two trains before coming to rest = $s_1 + s_2$ =122.5 + 200 = 322.5 m

Because the initial distance of separation is 500 m which is greater than 322.5 m, there will be no collision between the trains.

Problem : 24

In a car race, car A takes time t less than car B and passes the finishing point with a velocity v more than the velocity with which car B passes the point. Assuming that the cars start from rest and travel with constant accelerations a_1 and a_2, show that $\frac{v}{t} = \sqrt{a_1 a_2}$.

Sol. Let s be the distance covered by each car. Let the times taken by the two cars to complete the journey be t_1 and t_2, and their velocities at the finishing point be v_1 and v_2 respectively.

According to the given problem,

$v_1 - v_2 = v$ and $t_2 - t_1 = t$

Now, $\frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\sqrt{2a_1 s} - \sqrt{2a_2 s}}{\sqrt{\frac{2s}{a_2}} - \sqrt{\frac{2s}{a_1}}}$

$$= \frac{\sqrt{a_1} - \sqrt{a_2}}{\sqrt{\frac{1}{a_2}} - \sqrt{\frac{1}{a_1}}}$$

$$\therefore \frac{V}{t} = \sqrt{a_1 a_2}$$

***Problem : 25**

A particle moving along a straight line with initial velocity u and acceleration a continues its motion for n seconds. What is the distance covered by it in the last n^{th} second ?

Hint. $S = ut + \frac{1}{2}at^2$

Displacement in n seconds = $un + \frac{1}{2}an^2$

Displacement in (n –1) seconds

$$= u(n-1) + \frac{1}{2}a(n-1)^2$$

Displacement in n^{th} second = Displacement in n seconds – displacement in (n – 1) seconds.

$$\therefore S_n = u + a\left(n - \frac{1}{2}\right).$$

PROBLEMS BASED ON GRAPHS

Problem : 26

A bus accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds then, evaluate.

(a) the maximum velocity achieved and

(b) the total distance travelled graphically.

Sol. (a) Let t_1 be the time of acceleration and t_2 that of deceleration of the bus.

The total time is $t = t_1 + t_2$.

Let v_{max} be the maximum velocity.

As the acceleration and deceleration are constants the velocity time graph is a straight line as shown in the figure.with +ve slope for acceleration and -ve slope for deceleration.

From the graph,

the slope of the line OA gives the acceleration α.

$\therefore \alpha$ = slope of the line OA = $\frac{v_{max}}{t_1} \Rightarrow t_1 = \frac{v_{max}}{\alpha}$

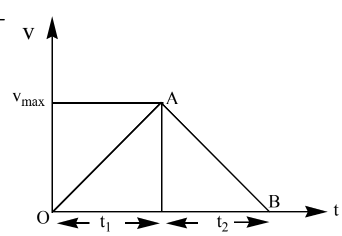
the slope of AB gives the deceleration β

$\therefore \beta$ = slope of AB = $\frac{v_{max}}{t_2} \Rightarrow t_2 = \frac{v_{max}}{\beta}$

$$t = t_1 + t_2 = \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta}$$

$$t = v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$\therefore v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)t$$



(b) Displacement = area under the v-t graph

= area of Δ OAB

$$= \frac{1}{2}(base)(height) = \frac{1}{2}t\,v_{max}$$

$$= \frac{1}{2}t\left(\frac{\alpha\beta t}{\alpha+\beta}\right)$$

$$= \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$$

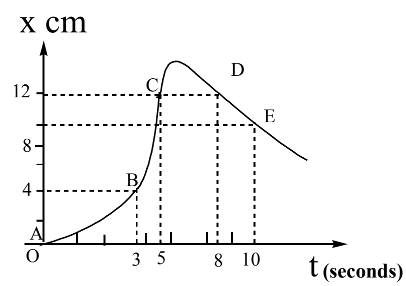
Problem : 27

Figure shows the motion of a particle along a straight line. Find the average velocity of the particle during the intervals

(a) A to E; (b) B to E; (c) C to E;

(d) D to E; (e) C to D.

Sol.



(a) As the particle moves from A to E, A is the initial point and E is the final point.

The slope of the line drawn from A to E

i.e., $\frac{\Delta x}{\Delta t}$ gives the average velocity during that interval of time.

The displacement Δx is

$x_E - x_A$ = 10 cm – 0 cm = +10 cm

The time interval $\Delta t_{EA} = t_E - t_A$ = 10s.

$\therefore$ During this interval average velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+10cm}{10s} = +1cms^{-1}$$

(b) During the interval B to E, the displacement

$\Delta x = x_E - x_B = 10cm - 4cm = 6cm$ and

$\Delta t = t_E - t_B = 10s - 3s = 7s.$

$\therefore$ Average velocity $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{6cm}{7s}$

= +0.857 cms^{-1} = 0.86cms^{-1}

(c) During the interval C to E,the displacement $\Delta x = x_E - x_C = 10cm - 12cm = 2cm$ and

$\Delta t = t_E - t_C = 10s - 5s = 5s$

$\therefore \bar{v} = \frac{\Delta x}{\Delta t} = \frac{-2cm}{5s} = -0.4cms^{-1}$

(d) During the interval D to E, the displacement

$\Delta x = x_E - x_D = 10cm - 12cm = -2cm$

and the time interval

$\Delta t = t_E - t_D = 10s - 8s = 2s$

$\therefore \bar{v} = \frac{\Delta x}{\Delta t} = \frac{-2cm}{2s} = -1cms^{-1}$

(e) During the interval C to D,the displacement $\Delta x = x_D - x_C = 12cm - 12cm = 0$

and the time interval $\Delta t = t_D - t_C = 8s - 5s = 3s$

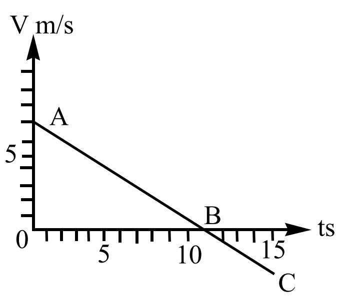
$\therefore$ The average velocity $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0m}{3s} = 0\ ms^{-1}$

(The particle has reached the same position during these 3s. The average velocity is zero because the displacement is zero).

***Problem : 28**

Velocity–time graph for the motion of a certain body is shown in Fig. Explain the nature of this motion. Find the initial velocity and acceleration and write the equation for the variation of displacement with time. What happens to the moving body at point B ? How does the body move after this moment ?

Sol.



The velocity – time graph is a straight line with–ve slope. The motion is uniformly retarding upto point B and there after uniformly accelerated upto C.

At point B the body stops and then its direction of velocity reversed.

The initial velocity at point A is $v_0 = 7\ ms^{-1}$.

$\therefore a = \frac{v_f - v_o}{\Delta t} = \frac{0 - 7ms^{-1}}{11s} = \frac{-7}{11} ms^{-2} = 0.64ms^2$

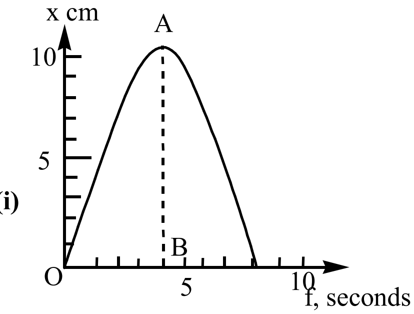
The equation of motion for this body which gives variation of displacement with time is $S = 7t - \frac{1}{2}0.64t^2$ $= 7t - 0.32t^2$.

***Problem : 29**

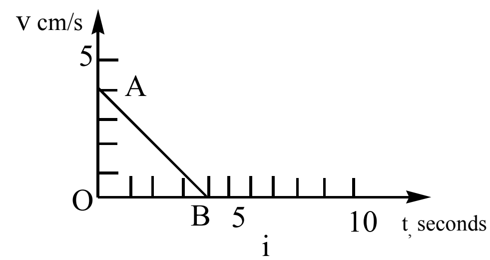
The graphs in (i) and (ii) show the S – t graph and V – t graph of a body.

Are the motions shown in the graphs represented by OAB the same ? explain

(i)



(ii)



Sol. **The motion shown by the two graphs are not same.**

i) In the given s – t graph OA, is a uniform retardation motion.

Here,

displacement = (average velocity) x (time)

$\therefore 10 = \left(\frac{u+0}{2}\right) \times 4$

$\therefore u = 5ms^{-1}$

using $v^2 - u^2 = 2as$

$0 - 5^2 = 2a(10)$

$a = -1.25\ ms^{-2}$

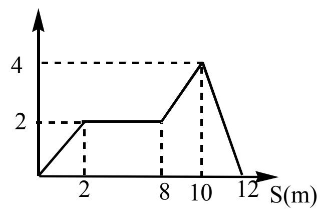
ii) In the given V – t graph, OA is a uniform retardation of motion

a = slope of the line $= \frac{OA}{OB} = \frac{-4ms^{-1}}{4s} = -1ms^{-2}$

Thus the two graphs even though represent uniform retardation motions, the magnitudes are not equal.

***Problem : 30**

The acceleration - displacement graph of a particle moving in a straight line is given as in the fig. The initial velocity of the particle is zero. Find the velocity of the particle when displacement of the particle is s = 12m.



Sol. From the equation $v^2 - u^2 = 2as$

$$as = \frac{v^2 - u^2}{2} = \text{area under a-s graph}$$

initial velocity u = 0;

$$\therefore as = \frac{v^2}{2} = \text{area under a-s graph.}$$

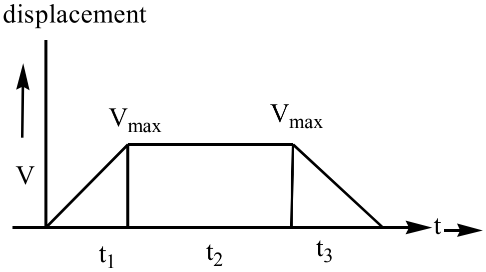
$$\therefore V = \sqrt{2(\text{area under } a - s \text{ graph})}$$

$$= \sqrt{\left(\frac{1}{2}(2)(2) + 6\times 2 + \frac{1}{2}(2+4)2 + \frac{1}{2}(2)4\right)2} = \sqrt{2\times 24} = 4\sqrt{3}ms^{-1}$$

***Problem : 31**

A body starts from rest and travels a distance S with uniform acceleration, then moves uniformly a distance 2S and finally comes to rest after moving further 5S under uniform retardation. Find the ratio of average velocity to maximum velocity :

Sol. **Graphically :** Area of (V–t) curve represent displacement



$$S = \frac{1}{2}V_{max}t_1 \text{ or } t_1 = \frac{2S}{V_{max}}$$

$$2S = V_{max}\, t_2 \text{ or } t_2 = \frac{2S}{V_{max}}$$

$$5S = \frac{1}{2}V_{max}t_3 \text{ or } t_3 = \frac{10S}{V_{max}}$$

$$V_{av} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$V_{av} = \frac{S + 2S + 5S}{\frac{2S}{V_{max}} + \frac{2S}{V_{max}} + \frac{10S}{V_{max}}}$$

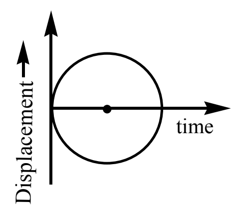
$$\frac{V_{av}}{V_{max}} = \frac{8S}{14S} = \frac{4}{7} \quad \text{(OR)}$$

$$\frac{V_{av}}{V_{max}} = \frac{\text{Total displacement}}{2\begin{pmatrix}\text{total displacement}\\ \text{during acceleration}\\ \text{andretardaton}\end{pmatrix} + \begin{pmatrix}\text{Displacement}\\ \text{During uniform}\\ \text{velocity}\end{pmatrix}}$$

$$\therefore \frac{V_{av}}{V_{max}} = \frac{8S}{2(S + 5S) + 2S} = \frac{8}{14} = \frac{4}{7}$$

Problem : 32

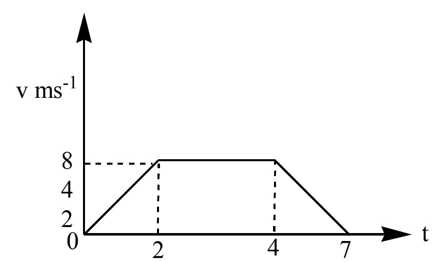
Figure given here shows the displacement time graph for a particle. Is it practically possible ? Explain.



Sol. From the graph, it is evident that, at any instant of time the particle possesses two displacements, which is impossible.

Problem : 33

Figure given here shows the variation of velocity of a particle with time.



Find the following :

i) Displacement during the time intervals.

a) 0 to 2 sec. b) 2 to 4 sec.and c) 4 to 7 sec

ii) Accelerations at

a) t = 1 sec, b) t = 3 sec. and c) t = 6 sec.

iii) Average acceleration

a) between t = 0 to t = 4 sec.

b) between t = 0 to t = 7 sec.

iv) Average velocity during the motion.

Hint.

i) displacement = Area enclosed between v – t graph and time axis.

ii) Acceleration = slope of v – t curve

iii) Average acceleration = $\frac{\text{Total change in velocity}}{\text{Total time}}$

iv) Average velocity = $\frac{\textit{Total displacement}}{\textit{Total time}}$

Ans. (i) a) 8m b) 16 m c) 12m

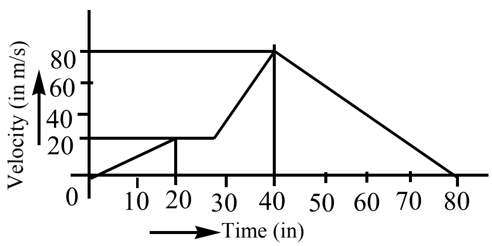
ii) a) $4ma^{-2}$ b) 0 c) $-2/3ms^{-2}$

iii) a) $2m/s^2$ *b) 0*

iv) $\frac{36}{7}ms^{-1}$

Problem :.34

The velocity time graph of a moving object is given in the figure. Find the maximum acceleration of the body and distance travelled by the body in the interval of time during which this acceleration exists.

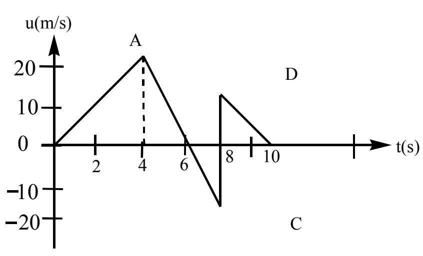


Sol. Acceleration is maximum when slope is maximum $a_{max} = \frac{80-20}{40-30} = 6m/s^2$

$S = 20\ m/s \times 10s + \frac{1}{2}\ x\ 6m/s^2 \times 100\ s^2 = 500\ m$

Problem : 35

The velocity–time graph of a body moving in a straight line is shown in Fig. Find the displacement and distance travelled by the body in 10 sec.



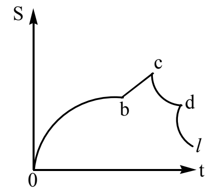
Hint. The area enclosed by velocity-time graph with time axis measures the displacement travelled in the given time.

Ans. $S = 50m^{-2}$

Problem : 36

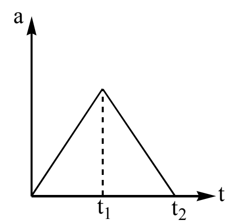
S –t group of a particle moving an a straight line is as shown. On which part the force acting is zero.

Sol : $v = \frac{ds}{dt} =$ constant in the part bc $\therefore a = 0$

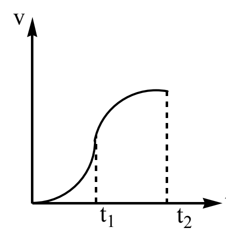


$\Rightarrow F = 0$ in the part bc

Note 18 : The acceleration, time graph is as shown



The corresponding v–t graph will be



Problem : 37

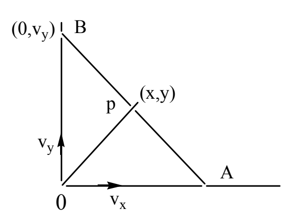
Three particles start from the origin at the same time, one with a velocity v_1 along x - axis , the second along the y - axis with a velocity v_2 and the third along x = y line. The velocity of the third so that the three may always lie on the same line is

Sol. Let time interval be chosen as 1 second.

$$\frac{PA}{PB} = \frac{OA}{0B} = \frac{v_x}{v_y}$$

So , P (x ,y) divides AB in the ratio $v_x : v_y$

$$x = \frac{v_x \times 0 + v_y \times v_x}{v_x + v_y} = \frac{v_x v_y}{u_x + v_y}$$



$$y = \frac{v_x v_y + v_y \times 0}{v_x + v_y} = \frac{v_x v_y}{v_x + v_y}$$

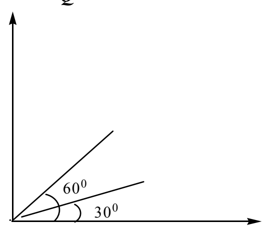
$$v = \sqrt{x^2 + y^2}$$

$$= \sqrt{2}\,\frac{v_x v_y}{v_x + v_y}$$

Now, repalce v_x by v_1 and v_y by v_2

Problem : 38

The displacement - time graphs of two particles P and Q are as shown in the figure. The ratio of their velocities V_p and V_Q will be

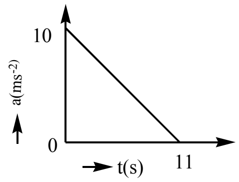


Sol. The velocity of a particle is equal to the slope of time - dispalcement straight line.

$$\frac{V_P}{V_Q} = \frac{\tan 30^0}{\tan 60^0} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = 1:3$$

Problem : 39

The a - t graph is shown in the figure. The maximum velocity attained by the body will be



Sol : Maximum velocity = at

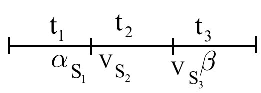
=Area between v-t graph and t-axis

$$= \frac{1}{2} \times 11 \times 10 = 55m$$

Problem : 40

A car travels starting form rest with constant acceleration α and reaches a maximum velocity V. It travels with maximum velocity for some time and retards uniformly at the rate of β and comes to rest. If s is the total distance and t is the total time of journey then t =

Sol :

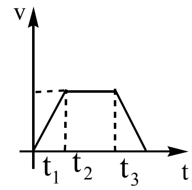


$$V = 0 + \alpha t, ; t_1 = \frac{v}{\alpha}$$

$$S_1 = \frac{1}{2}\alpha t_1^2 = \frac{v^2}{2\alpha}$$

$$O - V = \beta t_3;\ t_3 = \frac{v}{\beta}$$

$$O^2 - V^2 = -2\beta S_3$$



$$S_3 = \frac{v_2}{2\beta} = \frac{1}{2}\beta t_3^2$$

$$t_2 = \frac{S_2}{V} = \frac{S - (S_1 + S_3)}{V}$$

$$= \frac{S - \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)}{V}$$

$$t = t_1 + t_2 + t_3$$

$$= \frac{v}{\alpha} + \frac{s}{v} - \frac{v}{2\alpha} - \frac{v}{2\beta} + \frac{v}{\beta}$$

$$\boxed{t = \frac{s}{v} + \frac{v}{2}\left(\frac{\alpha + \beta}{\alpha\beta}\right)}$$

$$s_2 = s - (s_1 + s_2)$$

$$= s - \left(\frac{v^2}{2\alpha} + \frac{v^2}{2\alpha}\right)$$

$$= s - \frac{v^2}{\alpha}$$

$$= s - (vt - s)$$

$$\boxed{s_2 = 2s - vt}$$

[S_2= distance travelled with constant velocity]

PROBLEMS BASED ON CALCULUS

Problem : 41

A point moves the plane x - y according to the law $x = k \sin \omega t$ and $y = k(1 - \cos \omega t)$ where k and ω are positive constants. Find the distance s traversed by the particle during time t.

Sol. $\frac{dx}{dt} = v_x = k\omega \cos \omega t$

and $\frac{dy}{dt} = v_y = k\omega \sin \omega t$

Now, speed $v = \sqrt{\left(v_x^2 + v_y^2\right)} = k\omega = \text{constant}$

$\therefore\ s = vt = k\omega t$.

Problem : 42

The coordinates of a body moving in a plane at any instant of time t are $x = \alpha t^2$ and $y = \beta t^2$.

The velocity of the body is

Sol. $x = \alpha t^2 \Rightarrow v_x = \frac{dx}{dt} = 2\alpha t$

$y = \beta t^2 . \Rightarrow v_y = \frac{dy}{dt} = 2\beta t$

$\therefore velocity\ v = \sqrt{v_x^2 + v_y^2} \Rightarrow \sqrt{(2\alpha t)^2 + (2\beta t)^2}$

$= 2t\sqrt{\alpha^2 + \beta^2}$

Problem : 43

The motion of a particle along a straight line is described by the function $s = 6 + 4t^2 - t^4$ in SI units. Find the velocity, acceleration, at t=2s, and the average velocity during 3rd second.

Sol. $s = 6 + 4t^2 - t^4$

Velocity $= \frac{ds}{dt} = 8t - 4t^3$ *when* $t = 2$

Velocity = 8 × 2 -4 × 2^3

Velocity = 16 m/s

Acceleration $a = \frac{d^2 s}{dt^2} = 8 - 12t^2$ when t=2

acc = 8-12 × 2^2 = -40

acc = -40 m/s^2

displacement in 2 seconds

$s_1 = 6 + 4.\ 2^2 - 2^4 = 6$ m

displacement in 3 seconds

$s_2 = 6 + 4.\ 3^2 - 3^4 = -39$ m

displacement during 3rd second

$= s_2 - s_1 = -39 - 6 = -45$ m

$\therefore$ Average velocity during 3rd second

$= \frac{\pm 45}{1} = -45\ m/s$

-ve sign indicates that the body is moving in opposite direction to the initial direction of motion.

Problem : 44

A particle moves according to the equation $t = \sqrt{x} + 3$, where will be the particle come to the rest for the first time

Sol. $x = (t-3)^2$

$x = t^2 - 6t + 9$

$v = \frac{dx}{dt} = 2t - 6$

$0 = 2t - 6$

$\Rightarrow \boxed{t = 3s}$

Problem : 45

Acceleration of a particle is varying according to the law a = – ky. Find the velocity as a function of y, and intial velocity V_0

Sol. $a = \frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt}$

$\int_{V_0}^{V} v\,dv = \int_0^y -ky\,dy \Rightarrow V_0^2 - V^2 = ky$

$\boxed{V = \sqrt{V_0^2 - ky^2}}$

Problem : 46

The velocity of a particle moving in the positive direction of the X-axis varies as $V = K\sqrt{S}$ where K is a positive constant. Draw V – t graph.

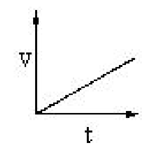
Sol. $V = K\sqrt{s}$

$\frac{dS}{dt} = K\sqrt{S}$ $\therefore \int_0^S \frac{dS}{\sqrt{S}} = \int_0^t K\,dt$

$\therefore 2\sqrt{S} = Kt$ and $S = \frac{1}{4}K^2 t^2$

$\Rightarrow V = \frac{dS}{dt} = \frac{1}{4}K^2 2t = \frac{1}{2}K^2 t$

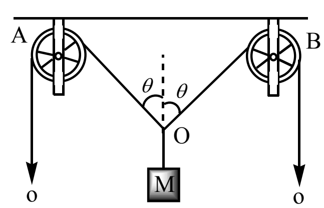
$\therefore V \propto t$



$\therefore$ The V – t graph is a striaght line passing through the origin

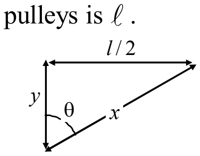
Problem : 47

In the arrangement shown in figure the ends of an inextinsible string move downwards with uniform speed u. Pulleys A and B are fixed. Find the speed with which the mass M moves upwards.



Sol. Suppose the distance of point O from the ceiling is y and the distance of point O from each pulley is x and the distance between the two pulleys is ℓ.

$x^2 = y^2 + \frac{l^2}{4}$



Differentiating the above eqation w.r.t to "*t*"

$2x\left(\frac{dx}{dt}\right) = 2y\left(\frac{dy}{dt}\right) + \frac{1}{4}2\ell\left(\frac{d\ell}{dt}\right)$

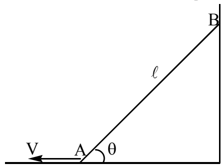
But, $u = -\frac{dx}{dt}$, $v = \frac{dy}{dt}$ and $\frac{d\ell}{dt}$

$\therefore -xu = yv$ and $v = -u\frac{x}{y} = u\sec\theta$

$\therefore$ Velocity of mass = $v = u\sec\theta$ (upwards)

Problem : 48

Figure shows a rod of length ℓ resting on a wall and the floor. Its lower end A is pulled towards left with a constant velocity v. Find the velocity of the other end B downward when the rod makes an angle θ with the horizontal.



Sol. In such type of problems, when velocity of one part of a body is given and that of other is required, we first find the relation between the two displacements, then differentiate them with respect to time. Here if the distance from the corner to the point A is x and up to B is y. Then

$v = \frac{dx}{dt}$ & $v_B = -\frac{dy}{dt}$

(–sign denotes that yis decreasing)

Further, $x^2 + y^2 = \ell^2$

Differentiating with respect to time t

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ $\qquad xv = yv_B$

$v_B = (v)\frac{x}{y} = v\cot\theta$

Problem : 49

Acceleration of a particle at any time t is $\vec{a} = \left(2t\hat{i} + 3t^2\hat{j}\right)m/s^2$. If initially particle is at rest, find the velocity of the particle at time t = 2s.

Sol. Here ***acceleration is a function of time,*** i.e., acceleration is not constant. So, we ***cannot apply*** $\vec{v} = \vec{u} + \vec{a}t$.

We will have to go for integration for finding velocity at any time t.

$a = \frac{d\vec{v}}{dt}$ Thus $d\vec{v} = \vec{a}\,dt$

or $\int_0^{\vec{v}} d\vec{v} = \int_0^2 \vec{a}\,dt$ or $\vec{v} = \int_0^2 \left(2t\hat{i} + 3t^2\hat{j}\right)dt$

$= \left[t^2\hat{i} + t^3\hat{j}\right]_0^2 = \left(4\hat{i} + 8\hat{j}\right)m/s$

Therefore, velocity of particle at time t = 2s is $\left(4\hat{i} + 8\hat{j}\right)m/s$

Problem : 50

A point moves the plane x – y according to the law x = k sin ωt and y = k (1–cos ωt) where k and ω are positive constants. Find the distance s traversed by the particle during time t.

Sol. $\frac{dx}{dt} = v_x = k\omega\cos\omega t$

and $\frac{dy}{dt} = v_y = k\omega\sin\omega t$

Now, speed $v = \sqrt{\left(v_x^2 + v_y^2\right)} = k\omega = \text{constant}$

$\therefore$ $s = vt = k\omega t$.

Problem : 51

$t = ax^2 + bx$ find acceleration ?
(a, b are constants)

Sol. $t = ax^2 + bx$

$\frac{dt}{dx} = 2ax + b$

$v = \frac{dx}{dt} = \frac{1}{2a + b}$

$\frac{dv}{dt} = \frac{-2a}{(2ax+b)^2}\frac{dx}{dt}$

$= \frac{-2a}{(2ax+b)3} = \boxed{-2av^3}$

Problem : 52

A point moves rectilinearly with deceleration whose modulus depends on the velocity v of the particle as $\alpha = k\sqrt{v}$, where k is a positive constant. At the initial moment the velocity of the point is equal to V_0. What distance will it take to cover that distance?

Sol. Let t_0 be the time in which it comes to a stop. Given that $-\frac{dv}{dt} = k\sqrt{V}$

$\int_{v_0}^{t_0} k\,dt = \int_{v_0}^{0} -\frac{dv}{\sqrt{v}}$

$\because kt_0 = 2\sqrt{v_0}$

$\therefore t_0 = \frac{2}{k}\sqrt{v_0}$

Now to find the distance covered before stopping,

$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$

But, $\frac{dv}{dt} = -k\sqrt{V}$;

$\therefore v\frac{dv}{ds} = -k\sqrt{V}$

$\therefore \sqrt{v}dv = -kds$

$\therefore \int_{V_0}^{0} \sqrt{v}dv = -\int_{V0}^{s} k\,ds \Rightarrow s = \frac{2}{3k}V_0^{\frac{3}{2}}$

Problem : 53

A particle moves according to the equation $\frac{dv}{dt} = \alpha - \beta v$ where α and β are costants. Find velocity as a function of time. Assume body starts from rest.

Sol. $\frac{dv}{dt} = \alpha - \beta v$

$\int_0^v \frac{dv}{\alpha - \beta v} = \int_0^t dt$

$\frac{\left[ln(\alpha - \beta v)\right]_0^V}{-\beta} = t$

$ln(\alpha - \beta v) - ln\,\alpha = -\beta t$

$ln\frac{\alpha - \beta v}{\alpha} = -\beta t$

$\frac{\alpha - \beta v}{\alpha} = e^{-\beta t}$

$1 - \frac{\beta}{\alpha}v = e^{-\beta t}$

$$\boxed{v = \frac{\alpha}{\beta}\left(1 - e^{-\beta t}\right)}$$

Problem : 54

The acceleration a of a particle depends on displacement S as a = S + 5. It is given that initially S = 0 and V = 5 m/s.
Find relation between i) V and S ii) S and t

Sol. i) : $a = \frac{dV}{dt} = \frac{dV}{dS}.\frac{dS}{dt} = S + 5$

$\therefore V\frac{dV}{ds} = S + 5$

$\therefore \int_5^V V\,dV = \int_0^S (S+5)dS$

$\Rightarrow \left[\frac{V^2}{2}\right]_5^V = \left[\frac{S^2}{2} + 5S\right]_0^S$

$\Rightarrow V = (S+5)$

ii) $V = S + 5$

$\therefore \frac{ds}{dt} = (S+5)$

$\therefore \int_0^S \frac{dS}{S+5} = \int_0^t dt$

$\therefore \log_e[S+5]_0^s = t$, $\log_e\left(\frac{S+5}{5}\right) = t$

Problem : 55

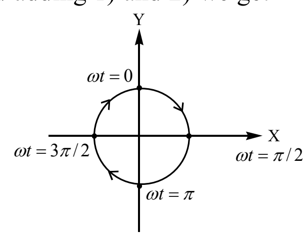
The x and y co-ordinates of a particle are $x = A\sin(\omega t)$ and $y = A\sin(\omega t + \pi/2)$. Find the motion of the particle

Sol. Given $x = A\sin\omega t$

$y = A\sin(\omega t + \pi/2) = A\cos\omega t$

squaring and adding 1) and 2) we get

$x^2+y^2=A^2$



i.e . path of the particle is a circle with centre at origin and radius A

At time $\omega t = 0$ x = 0 and y=A

and at $\omega t = \pi/2\, x = A$ and y=0

at $\omega t = \pi\, x = 0$ y = –A and so on.

$\therefore$ The motion is circular clockwise

Problem : 56

An object is projected in X – Y plane in which velocity changes according to relation $\vec{V} = a\hat{i} + bx\hat{j}$. Equation of path of particle is:

a) Hyperbolic **b) Circular**
c) Elliptical **d) Parabolic**

Sol. $\frac{dx}{dt} = a$

x = at

$V_y = bx = b\,at$

$\frac{dy}{dt} = b$ at

$\int dy = \int bat\,dt$

$y = \frac{bat^2}{2}$

$y = \frac{ba}{2}\frac{x^2}{a^2} = \frac{bx^2}{2a}$

$y \propto x^2$ i.e., parabolic.

Problem : 57

The displacement (x) of a particle varies with time as $x = ae^{-\alpha t} + be^{\beta t}$ where a, b, α, β are positive constant. Find how does the velocity of particle change with time,

Sol : $x = ae^{-\alpha t} + be^{\beta t}$

$$V = \frac{dx}{dt} = -\alpha ae^{-\alpha t} + \beta be^{\beta t}$$

$$V = \beta be^{\beta t} - \alpha ae^{-\alpha t}$$

$\therefore$ V increases as t increases.

***Note 1**:* In the case of a vertically projected body, velocity is maximum at projection point and minimum (zero) at highest point

***Note 2** :* Velocity goes on decreasing .

Note 3: Velocity at any point during the upward journey = velocity at the same point during the downward journey (numerically)

Thus projection velocity = Striking velocity (numerically) .

Note 4: Change in velocity in the entire journey = 2u $\{(\Delta v = v_f - v_i = u - (-u) = 2u\}$

Note 5: Similarly Change in momentum. during the entire journey = 2mu

Note : 6

Velocity at half maximum height = $\frac{u}{\sqrt{2}}$

Sol. using the equation $v^2-u^2=2as$, we have a=-g ,

$s = \frac{1}{2}H = \frac{1}{2}\frac{u^2}{2g}$, we get

$$v^2 - u^2 = 2(-g)\times\frac{u^2}{4g} \Rightarrow v^2 - u^2 = \frac{-u^2}{2} \Rightarrow v^2 = \frac{u^2}{2} \Rightarrow v = \frac{u}{\sqrt{2}}$$

Note 7:

Velocity at $\frac{3}{4}th$ of maximum height = $\frac{u}{2}$

Sol. using the equation $v^2-u^2=2as$, we have a=-g ,

$s = \frac{3}{4}H = \frac{3}{4}\frac{u^2}{2g}$, we get

$$v^2 - u^2 = 2(-g)\times\frac{3u^2}{8g} \Rightarrow v^2 - u^2 = \frac{-3u^2}{4} \Rightarrow v^2 = \frac{u^2}{4} \Rightarrow v = \frac{u}{2}$$

Note 8 : Distance covered by a body projected vertically up in the 1st second of its upward journey = Distance covered by it in the last second of its entire journey = $u - \frac{g}{2}$

Sol. we know that for a body projected vertically up,

$s_n = u - g\left(n - \frac{1}{2}\right)$.

Substituting n=1 gives $s_1 = u - \frac{g}{2}$ (clearly , the distance travelled in last second is same as that of 1st second)

Note 9:

Distance covered by a body projected vertically up in the last one second of its upward journey = Distance covered by it in the 1st second of its downward journey = $\frac{g}{2}$

Sol. For a body falling downwards , we know that

$s = \frac{1}{2}gt^2$

Substituting t=1 gives us $s = \frac{g}{2}$ (clearly , the distance travelled in 1st second in the downward journey is same as that of last second of upward journey)

Note 10:

Time taken by vertically projected up body to reach $\frac{3}{4}th$ of maximum height = $\frac{t_a}{2}$

Sol. we know that , for a body projected vertically up v=u-gt $\Rightarrow \frac{u}{2} = u - gt$ (since at $\frac{3}{4}th$ of maximum height , velocity = $\frac{u}{2}$)

$$\Rightarrow gt = \frac{u}{2} \Rightarrow t = \frac{u}{2g} \Rightarrow t = \frac{1}{2}\times t_a$$

Note 11:

A body projected vertically up takes a time $t = t_a\left(1 - \frac{1}{\sqrt{2}}\right)$ to reach half of maximum height .

Solution : we know that , for a body projected vertically up v=u-gt . $\Rightarrow \frac{u}{\sqrt{2}} = u - gt$ (since at half of maximum height , velocity = $\frac{u}{\sqrt{2}}$)

$$u-\frac{u}{\sqrt{2}}=gt \Rightarrow t=\frac{u}{g}\left(1-\frac{1}{\sqrt{2}}\right) \Rightarrow t=t_a\left(1-\frac{1}{\sqrt{2}}\right)$$

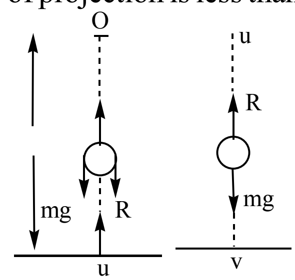
Note : 12

When air resistance is taken into account

i) Time of ascent is less than that in vacuum

ii) Time of ascent is less than time of descent

iii) The speed of the body when it reaches the point of projection is less than the speed of projection



F = mg + R   F = mg – R

$a = g + \frac{R}{m}$ $a^1 = g - R/m$

$0 = u - at_a$ $v = 0 + a^1 t_d$

$$t_a = \frac{u}{g + R/m}$$ $$v = g - \left(\frac{R}{m}\right)t_d$$

$$h = \frac{1}{2}\left(g + \frac{R}{m}\right)t_a^2 = \frac{1}{2}\left(g - \frac{R}{m}\right)t_d^2$$

$$\frac{t_a}{t_d} = \sqrt{\frac{g - R/m}{g + R/m}} = \frac{v}{u}$$

$$\therefore \frac{v}{u} = \frac{g - R/m}{g + R/m} \times \frac{t_d}{t_d}\sqrt{\frac{g - R/m}{g + R/m}}$$

For dropped bodies

i) Same resistance force R

$\Rightarrow a = g - R/m$

If m is more a is more

$\Rightarrow$ heavier body falls first

ii) If R is proportional to m then acceleration is same for both

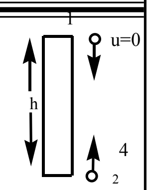
$\Rightarrow$ both the balls fall simulateneously in the same time

iii) If m is same R is less for smaller body; a = g–R/m is more for smaller body

$\Rightarrow$ smaller body falls first

App : 1

Body 1 is released from the top of a tower . At the same instant, body 2 is projected vertically up as shown then



a) height at which they meet is $t = \frac{h}{u}$

Sol. Let the two meet after a time 't' seconds then the in this the distance covered by both must be equal to height of true

i.e , $S_1+S_2=h$

$$\Rightarrow \frac{1}{2}gt^2 + ut\frac{1}{2}gt^2 = h \Rightarrow ut = h$$

$$\Rightarrow t = \frac{h}{u}$$

b) the time after which their velocities are equal is $t = \frac{u}{2g}$

Sol : Let the velocities be equal after a time 't'

$$\Rightarrow v_1 = v_2$$

$$\Rightarrow gt = u - gt \Rightarrow u \Rightarrow t = \frac{u}{2g}$$

c) Ratio of distances covered when the magnitudes of their velocities are equal is

$S_1 : S_2 = 1:3$

Sol : From above , velocities are equal after a time

$t = \frac{u}{2g}$ in this time

$$S_1 = \frac{1}{2}g\left(\frac{u}{2g}\right)^2 = \frac{1}{2}\times g \times \frac{u^2}{4g^2}$$

$$\Rightarrow S_1 = \frac{u^2}{8g}$$

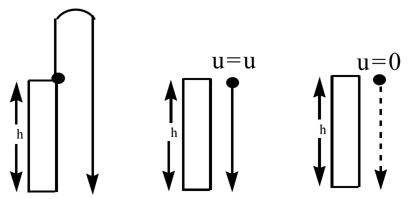
$$\Rightarrow S_2 = ut - \frac{1}{2}gt^2 = u\left(\frac{u}{2g}\right) - \frac{1}{2}g\left(\frac{u}{2g}\right)^2$$

$$= \frac{u^2}{2g} - \frac{u^2}{8g} = \frac{3u^2}{8g} \Rightarrow \mathbf{S_1 : S_2 = 1:3}$$

App: 2

Three bodies are projected from towers of same height as shown. 1st one is projected vertically up with a velocity 'u'. The second one is thrown down vertically with the same velocity and the third one is dropped as a freely falling body. If t_1, t_2, t_3 are the times taken by them to reach ground, then,

a) velocity of projection is $u = \frac{1}{2}g(t_1 - t_2)$



Sol. Clearly the extra time taken by the 2nd body is equal to the time of flight of 1st body

i.e, $t_1 - t_2 = \frac{2u}{g} \Rightarrow u = \frac{1}{2}g(t_1 - t_2)$

b) height of the tower is $h = \frac{1}{2}gt_1t_2$

Sol. We know that , for a vertically projected up body

$$s = ut - \frac{1}{2}gt^2$$

$\Rightarrow h = ut - \frac{1}{2}gt^2$ [similar to $ax^2+bx+c=0$]

The product of these far roots of their chapton gives

$$\mathbf{h = \frac{1}{2}gt_1t_2}$$

c) The time taken free fall in the 3rd case is given by $t = \sqrt{t_1t_2}$ $\quad t = \sqrt{\frac{2h}{g}}$

in above , the form that

$$h = \frac{1}{2}gt_1t_2 \Rightarrow t_1t_2 = \frac{2u}{g}$$

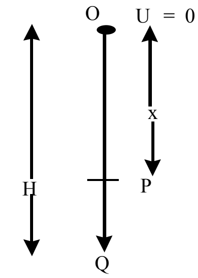
$$t_{free} = \sqrt{\frac{2h}{g}} = \sqrt{t_1t_2}$$

App: 3

A body falls freely from a height 'H'. After t seconds of fall, gravity casses to act. Find the time of flight

Sol. Let the total time taken be T

Let gravity cease at P



i,e . at $P, g = 0 \Rightarrow v = cons\tan t$

Step a] Distance converd in t second is $\frac{1}{2}gt^2 = x$

remaining distance PQ is to be convered by the body with constant velocity for which have $S_{PQ} = V_P \times t'$

Step b] Velocity at P is $V_P = gt$

Step c] $S_{PQ} = V_P \times t'$ (t'=time taken to cover the distance PQ)

$$\therefore t' = \frac{S_{PQ}}{V_P} = \frac{H-x}{gt}$$

$$\therefore t = \frac{H - \frac{1}{2}gt^2}{gt} = \frac{H}{gt} - \frac{t}{2}$$

iv) total time of fall is

$$T = t + t' = \left\{t + \frac{H}{gt} - \frac{t}{2}\right\}$$

App: 4

A particle is projected vertically upwards and it reaches the maximum height H in time T seconds. The height of the particle at any time t will be.

$$\left[H - \frac{1}{2}g(t-T)^2\right]$$

Sol. From v=u+gt

o = u-gt

u = gT i

$$H = \frac{u^2}{2g} = \frac{1}{2}gT^2 ......ii$$

Let h be the distance travelled in time t.

Then $h = ut - \frac{1}{2}gt^2$

$$h = gTt - \frac{1}{2}gt^2 ....iii$$

Subtracting (ii) from (iii)

$$h - H = gTt - \frac{1}{2}gt^2 - \frac{1}{2}gT^2$$

$$-\frac{g}{2}\left(-2Tt + t^2 + T^2\right) = -\frac{g}{2}\left(T - t\right)^2$$

$$\therefore h = H - \frac{1}{2}g\left(T - t\right)^2$$

App: 5

Roket is fired vertically up with an acceleration a. Fuel is exhausted after t sec. Maximum height it can reach is given by $\left[\frac{1}{2}at^2\left(1 + \frac{a}{g}\right)\right]$

Sol. **Step I :-** Distance covered by the rocket in t sec. (till fuel is exhausted)

$$S_1 = \frac{1}{2}at^2$$

Step II : Velocity at the end of t sec,

v=u+at = 0+at = at (∴ u = 0 for a rocket)

Step III:- Further distance it can go up after t sec.

(after fuel is exhausted) is given by

stopping distance $S_2 = \frac{(velocity)^2}{2 \times retardation}$ =

$$\frac{v^2}{2g} = \frac{(at)^2}{2g} = \frac{a^2t^2}{2g}$$

Step -IV = Maximum height the rocket car reach is , H= S_1 +S_2

$$H = \frac{1}{2}at^2 + \frac{1}{2}\frac{a^2t^2}{g} = \frac{1}{2}at^2\left(1 + \frac{a}{g}\right)$$

Note : 13

For a Vertically thrown up body, maximum height H = $\frac{1}{8}gT^2$ where 'T' is the time of flight

Sol. We know that time of flight $T = \frac{2u}{g}$

$$\Rightarrow u = \frac{gT}{2}$$

∴ Maximum height

$$H = \frac{u^2}{2g} = \frac{\left(\frac{gT}{2}\right)^2}{2g} = \frac{1}{8}gT^2$$

Similarly the same formula is applicable even in the case of a projectile.

App : 6

After falling for t_1 sec, a stone hits a horizontal glass plate, where it looses 50% of its velocity. It then takes t_2 sec to reach ground. Find the Height of the glass plate above the ground.

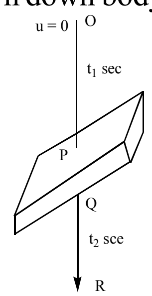
Sol. Let P be the upper edge and Q be the lower edge of the glass plate, as shown.

Step -I :- Velocity of the stone at P, $V_p = gt_1$

Step -II:- Velocity of the stone at Q, $v_Q = \frac{gt_1}{2}$

(∵ it looses 50% of velocity)

Step - III: from Q to R, the stone travels as a vertically thrown down body with an initial velocity $= \frac{gt_1}{2}$



Step -IV : Hence using the equation,

s=ut+$\frac{1}{2}at^2$

Where S = QR ; $u = v_Q = \frac{gt_1}{2}; a = +g$

t = t_2, we have,

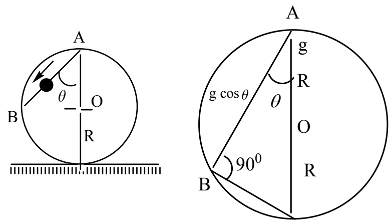
height of the glass plate above the ground =

$QR = \frac{gt_1}{2}t_2 + \frac{1}{2}gt_2^2$. From this equation, t_2 can be found.

App: 7

A frictionless wire is fixed between A and B inside a sphere of radius R . A small ball slips

along the wire.Find the time taken by the ball to slip from A to B



Sol. $S=\frac{1}{2}at^2$

i.e. AB $=\frac{1}{2}(g\cos\theta)t^2=2R\cos\theta$

$\therefore t=2\sqrt{\frac{R}{g}}$

App : 8

A particle projected vertically up from the top of a tower takes t_1 seconds to reach the ground. Another particle thrown downwards with the same velocity from the same point takes t_2 seconds to reach the ground. Then

a) Velocity of projection $=u=\frac{g}{2}(t_1-t_2)$

b) Height of tower = $H=\frac{1}{2}g(t_1t_2)$

c) A body dropped from the top tower takes times $t=\sqrt{t_1t_2}$ to reach the ground

$H=\frac{1}{2}g(t_1t_2)=\frac{1}{2}g\,t^2$

$\therefore t=\sqrt{t_1t_2}$

Hint.

For the body projected vertically upwards,

$H=\frac{1}{2}gt_1^2-ut_1$ ------- (1)

For the body projected vertically downwards

$H=ut_2+\frac{1}{2}gt_2^2$ -------- (2)

Solve (1), (2)

App: 9

A body projected vertically upwards from ground is at the same height h from the ground at two instants of time t_1 and t_2 (both being measured from the instant of projection) Now

a) $\mathbf{h=\frac{1}{2}g(t_1t_2)}$

b) Velocity of projection = $\mathbf{u=\frac{1}{2}g(t_1+t_2)}$

c) $H_{max}=\mathbf{\frac{1}{8}g(t_1+t_2)^2}$

d) A body dropped from height h takes time $\sqrt{t_1t_2}$ to reach the ground

Problem : 58

A body is projected vertically up with velocity u from a tower. It reaches the ground with velocity nu. The height of the tower is $H=\frac{u^2}{2g}(n^2-1)$

Sol. $v^2-u^2=2as$

Here u = u, v = nu, a = – g, s = – H

$(nu)^2-u^2=2gH$

$(n^2-1)u^2=2gH$ $\therefore H=\frac{u^2}{2g}(n^2-1)$

Problem : 59

Two bodies begin to fall freely from the same height. The second body begins to fall 'τ' s after the first. After what time from the begining of first body dose the distance between the bodies equals to ℓ ?

Sol. Let the time of fall of the 1st body be t seconds . Time of fall of second body = $t-\tau$.

Distances of free fall of the bodies in the above time intervals respectively are

$H_1=\frac{gt^2}{2};H_2=\frac{g(t-\tau)^2}{2}$

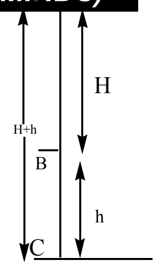
Therefore $\ell=H_1-H_2=gt\tau-\frac{1}{2}g\tau^2$

$\therefore t=\frac{\ell}{g\tau}+\frac{\tau}{2}$

Problem : 60

One body falls freely from a point A at a height (H + h) while another body is projected upwards with an initial velocity V_0 from point C at the same time as the first body begins to fall. What should be the velocity V_0 of the second body so that the bodies meet at a point B at the height 'h'? What is the maximum height attained by 2nd body for the given initial velocity? What is the value of V_0 if H = h ?

Sol.



i) Suppose the two bodies meet after t seconds.

$$H = \frac{gt^2}{2} \quad .....(1)$$

$$h = V_o t - \frac{gt^2}{2} \quad .... (2)$$

Solving (1) & (2)

$$V_o = (H+h)\sqrt{\frac{g}{2H}}$$

ii) We know that $H_{Max} = \frac{u^2}{2g}$

$$H_{\max} = \frac{V_o^2}{2g} = \frac{(H+h)^2}{2g}\frac{g}{2H} = \frac{(H+h)^2}{4H} \; (Here\ H_{\max} > h)$$

iii) When H = h, we get $V_o = \sqrt{2gh}$.

Problem : 61

For a freely falling body, Find the ratio of the times taken to fall successive equal distances.

Ans. Ratio of times taken to fall equal distances is

$$(\sqrt{1}-\sqrt{0}) : (\sqrt{2}-\sqrt{1}) : (\sqrt{3}-\sqrt{2}) : .... : (\sqrt{n}-\sqrt{n-1})$$

Hint. use $t = \sqrt{\frac{2h}{g}}$

Problem : 62

If a freely falling body covers half of its total distance in the last second of its journey, Find its time of fall.

Sol. Suppose t is the time of free fall.

$$h = \frac{1}{2}gt^2 \quad .... (1)$$

$$\frac{h}{2} = \frac{1}{2}g(t-1)^2 \quad .... (2)$$

Solving 1, 2

$t = (2+\sqrt{2})s$

since $2-\sqrt{2}$ is not acceptable.

Problem : 63

A balloon starts from rest, moves vertically upwards with an acceleration g/8 ms^{-2} . A stone falls from the ballon after 8s from the start. Find the time taken by the stone to reach the ground ($g = 9.8ms^{-2}$)

Sol. Step–1 : To find the distance of the stone above the ground about which it begins to fall from the balloon.

$$S = ut + \frac{1}{2}at^2$$

here, s = h , u = 0 , a = g/8

$$h = \frac{1}{2}\left(\frac{g}{8}\right)8^2 = 4g$$

Step–2 : The velocity of the balloon at this height can be obtained from v = u + at

$$V = 0 + \left(\frac{g}{8}\right)8 = g$$

This becomes the intital velocity ($u^{|}$) of the stone as the stone falls from the balloon at the height h.

$$\therefore u^{|} = g$$

Step-3 : For the total motion of the stone $h = \frac{1}{2}gt^2 - u^{|}t$

Here, h = 4g , $u^{|}$ = g, t = time of travel of stone.

$$\therefore -4g = gt - \frac{1}{2}gt^2$$

$$\therefore t^2 - 2t - 8 = 0$$

solving for 't' we get t=4 and – 2s. Ignoring negative value of time, t=4 s

Problem : 64

A rocket is fired upwards vertically with a net acceleration of $4m/s^2$ and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. There after it accelerates downwards with acceleration g and return back to ground.

i) Plot velocity – time graph for complete journey

ii) Displacement–time graph for the complete journey.

(Take g = 10 m/s².)

Sol. Stage i : To find velocity of rocket after 5 seconds

$$V_A = 0 + at_{OA} = (4)(5) = 20ms^{-1}$$

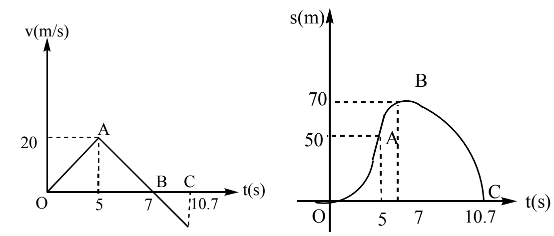
Stage ii : To find further time of ascent after 5 seconds.

$0 = 20 - g\,t_{AB}$

$$\therefore t_{AB} = \frac{20}{10} = 2\text{seconds}$$

Here, the total vertical displacement of stage i) and stage ii) is

= area of OAB = $\frac{1}{2}(7)(20)$ = 70m.



Stage –iii : If t_{BC} is time of descent then

$70 = \frac{1}{2}(10)\, t_{BC}^2 \qquad \therefore t_{BC} = \sqrt{14} = 3.7s$

Note 4.36 : $t_{OABC} = 7 + 3.7 = 10.7s$

Note 4.37 : S_{OA} = area under v – t graph

$= \frac{1}{2}(5)(20) = 50m$

Problem : 65

A stone is allowed to fall from the top of a tower 300 m heigh and at the same time another stone is projected vertically up from the ground with a velocity 100 ms^{-1}. Find when and where the two stones meet ?

Sol. Suppose the two stones meet at a height x from ground after t seconds.

$x = 100t - \frac{1}{2}gt^2$ .....(1) $\qquad 300 - x = 0 + \frac{1}{2}gt^2$ ....(2)

Solve 1, 2

t = 3 sec, x = 255.9m

Problem : 66

Ball A is dropped from the top of a building and at the same instant that a ball B is thrown vertically upward from the ground. When the balls collide, they are moving in opposite directions and the speed of A is twice the speed of B. At what fraction of the height of the building did the collision occur ?

Sol. Given $V_A = 2V_B$

Let h_1 and h_2 are the distances travelled by the two balls

$\therefore \sqrt{2gh_1} = 2\sqrt{u^2 - 2gh_2}$

$\therefore h_1 + 4h_2 = \frac{2u^2}{g}$ .... (1)

If they meet after t seconds, for the condition $V_A = 2V_B$

0 + gt = 2 (u – gt)

$\therefore gt = 2u - 2gt \Rightarrow 2u = 3gt$

$\therefore t = 2u/3g$

Also $h_1 + h_2 = \frac{1}{2}gt^2 + \left(ut - \frac{1}{2}gt^2\right) = ut$

$\therefore h_1 + h_2 = ut = u\left(\frac{2u}{3g}\right) = \frac{2u^2}{3g}$ ....(2)

Solving (1) & (2) we get $h_1 = \frac{2u^2}{9g}; h_2 = \frac{4u^2}{9g}$

$\therefore \frac{h_1}{h_2} = \frac{2u^2/9g}{4u^2/7g} = \frac{1}{2}$

Problem :67

An object falls from a bridge which is 45 m above the water. It falls directly into a small row – boat moving with constant velocity that was 12m from the point of impact when the object was released. What was the speed of the boat ?

Sol. Velocity of boat = $V = \frac{s}{t}$

Here,

s = 12m

t = time of fall of object from bridge

$= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 45}{10}} = 3s$

$\therefore V = \frac{12}{3} = 4ms^{-1}$

Problem : 68

Two balls are dropped to the ground from different heights. One ball is dropped 2s after the other, but both strike the ground at the same time 5s after the Ist is dropped.

a) What is the difference in the heights from which they were dropped ?

b) From what height was the first ball dropped?

Sol. a) For the first ball s = h_1, u = 0, t = 5 s

$\therefore h_1 = 0 \times 5 + \frac{1}{2}9.8x5^2 = 122.5m$

For the second ball s = h_2, u = 0, t = 3s

$\therefore h_2 = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 9 = 4.9 \times 9 = 44.1m$

Difference in heights

$h = h_2 - h_1 = 122.5 - 44.1 = 78.4m$

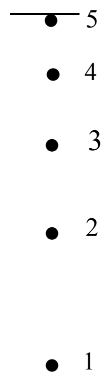
b) The first ball was dropped from a height of

$h_1 = 122.5m$

Problem : 69

Drops of water fall at regular intervals from the roof of a building of height H = 16m, the first drop striking the ground at the same moment as the fifth drop detaches from the roof. Find the distance between the successive

drops.



Sol. Step - i : Time taken by the first drop to touch the ground = $t=\sqrt{\frac{2h}{g}}$

For h = 16m, $t=\sqrt{\frac{2}{g}}$.

Time interval between two drops is

$$t_{interval}\left(\frac{1}{n-1}\right)t=\left(\frac{1}{4}\right)t=\frac{1}{4}\sqrt{\frac{2}{g}}$$

Where n = number of drops.

Step - ii :

Distance between first and second drops

$$=S_1-S_2=\frac{1}{2}gt_{interval}^2\left[4^2-3^2\right]=7\text{m}.$$

Distance between second and third drops

$$=S_2-S_3=\frac{1}{2}gt_{interval}^2\left[3^2-2^2\right]=5\text{m}.$$

Distance between third and fourth drops

$$=S_3-S_4=\frac{1}{2}gt_{interval}^2\left[2^2-1^2\right]=3\text{m}$$

Distance between fourth and fivfth drops.

$$=S_4-S_5=\frac{1}{2}gt_{interval}^2\left[1^2-0\right]=1\text{m}.$$

Problem : 70

i) How long does it take a brick to reach the ground if dropped from a height of 65m ? ii) What will be its velocity just before it reaches the ground ?

Hint. i) $t=\sqrt{\frac{2h}{g}}$ ii) $V=\sqrt{2gh}$

Ans. i) 3.64 s. ii) 35.69ms^{-1}

Problem : 71

A helicopter is ascending vertically with a speed of 8.0 ms^{-1}. At a height of 120 m above the earth, a packat is dropped from a window. How much time does it take for the package to reach the ground ?

Sol. Hint : $h=\frac{1}{2}gt^2-ut$

Ans. t = 5.83s

Problem : 4.72

If an object reaches a maximum vertical height of 23.0 m when thrown vertically upward on earth how high would it travel on the moon where the acceleration due to gravity is about one sixth that on the earth ? Assume that initial velocity is the same.

Hint. $H\propto\frac{1}{g}$

Ans. 138m

Problem : 73

An elevator ascends with an upward accleration of 0.2m/s^2. At the instant its upward speed is 3m/sec a loose bolt 5m high from the floor drops from the celing of the elevator. Find the time untill the bolt strikes the floor and the displacement it has fallen

Sol. Intial velocity of bolt relative to the floor of tyhe elevator = 0

acceleration of bolt relative to the floor of the elevator = (9.8 + 0.2) = 10ms^2

If t is time of the descent then $5=\frac{1}{2}\times10\times t^2$

$\therefore$ t = 1 second

If s is the displacement then

$s=\frac{1}{2}gt^2-ut$. $\therefore s=1.9m$

Problem : 74

A balloon is rising vertically upwards with uniform acceleration 15.7 m/s^2 .A stone is dropped from it. After 4s another stone is dropped from it. Find the distance between the two stones 6 s. after the second stone is dropped

Sol. if 'f' is upward acceleration of the balloon then the acceleration of the stones relative to the balloon is (f+g).

The initial velocity of each stone with respect to the balloon is zero.

Let s_1 and s_2 be the distances of the two stones from the balloon after 10s and 6 s respectively. Now

$$s_1=\frac{1}{2}(f+g)(10)^2=\frac{1}{2}(15.7+9.8)100=25.5\times50$$

$$s_2=\frac{1}{2}(25.5)(6)^2=25.5\times18$$

$$s_1-s_2=25.5\times32=816m$$

Problem : 75

A body falls freely from a height of 25m (g=10m/s^2) after 2sec gravity ceases to act Find the time taken by it to reach the ground?

Sol. 1) Distance covered in 2s under gravity

$$s_1=\frac{1}{2}gt^2=\frac{1}{2}(10)2^2=20m$$

velocity at the end of 2s

V = gt = (10)2 = 20m/s.

Now at this instant gravity ceases to act

$\Rightarrow$ velocity by here after becomes constant.

The remaining distance which is 125–20=105 m is covered by the body with constant velocity of 20m/s.

Time taken to cover 105 m with constant velocity is given by,

$t_1 = \frac{S}{V} \Rightarrow t_1 = \frac{105}{20} = 5.25s$

Hence total time taken to reach the ground

= 2 + 5.25 = 7.25 s

Problem : 76

A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enters water. Upto what ?

depth will the ball go? Howmuch time will it take to come again to the water surface Negiect air resistance and vescosity effects in water ($g = 9.8\ m/s^2$).

Sol : Velocity at the surface of water

$v = \sqrt{2gh} = \sqrt{2\times9.8\times19.6} = 19.6 m/s$

Acceleration of a body of density d_b in the liquid medium of density is

$g' = g\left(1-\frac{d_\ell}{d_b}\right) = g\left(1-\frac{d_\ell}{d_\ell/2}\right) = -g$

Using $v^2-u^2 = 2as$ in the water

$0 - (19.6)^2 = 2(-g)h$

h=19.6 m.

Using $s = ut + \frac{1}{2}at^2$, in the water when the ball returns to the surface, s = o

$0 = 19.6 + \frac{1}{2}(-9.8)t \qquad (\therefore a = -g)$

t = 4 s.

Probelem : 77

A parachutist drops freely from an aeroplane for 10 seconds before the parachute opens out. Then he descends with a net retardation of 2 m/sec². His velocity when he reaches the ground is 8 m/sec. Find the height at which he gets out of the aeroplane ?

Sol : Distance he falls before the parachute opens

is $\frac{1}{2}g\times100 = 490\ m$

Then his velocity = gt=98.0 m/s = u

Velocity on reaching ground = 8 = ϑ

retardation = 2

$\vartheta^2 - u^2 = 2as$

$8^2 - (98)^2 = 2\times(-2)S$

$\therefore S = \frac{106\times90}{4} = 2385\ m$

Total distance = 2385 + 490

=2875 m = height of aeroplane

Problem : 78

A stone is dropped into a well and the sound of splash is heard after 5.3 sec. If the water is at a depth of 122.5 m from the ground, the velocity of sound in air is

Sol : If t_1 is the time taken by stone to reach the ground and t_2 the time taken by sound to go up, then $t_1 + t_2 = 5.33$

Since $s = ut + \frac{1}{2}at^2$

$122.5 = 0t + \frac{1}{2}\times9.8\times t_1^2$

$\therefore t_1^2 = \frac{245}{9.8} = \frac{2450}{98} = 25$

$\therefore t_1 = 5s$

$\therefore t_2 = 0.33s$

Velocity of sound = $\frac{displacement}{time}$

$= \frac{122.5}{0.33} = 367\ m/s$

Problem : 79

A body is thrown vertically up with a velocity of 100 m/s and another one is thrown 4 sec after the first one. How long after the first one is thrown will they meet?

Sol : Let them meet after t sec.

$S_1 = 100t - \frac{1}{2}gt^2 \ and \ S_2 = 100(t-4) - \frac{1}{2}g(t-4)^2$

$\therefore 100t - \frac{1}{2}gt^2 = 100(t-4) - \frac{1}{2}g(t-4)^2$

$\therefore 400 = \frac{1}{2}g\left[t^2 - (t-4)^2\right] = \frac{1}{2}g.4(2t-4)$

$\therefore 2t - 4 = \frac{800}{4g} = 20, if \ g = 10 m/s^2$

$\therefore t = 12$ sec

Problem : 80

A lead ball is dropped into a lake from a diving board 20 m above the water. It hits the water with a certain velocity and then sinks to the bottom of the lake with the same velocity 6 sec after it is dropped. [g = 10 m/s²]. Find the depth of the lake.

Ans : 80 m

Sol : Velocity on reaching water $= \sqrt{2\times10\times20} = 20\ ms^{-1}$

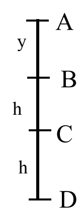
Time taken to reach water $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times20}{10}} = 2\,\text{sec}$

$\therefore$ depth of water (s) = vt = 20 (6-2) = 80 m

Problem : 81

A particle is dropped from point A at a certain height from ground . It falls freely and passes through three points B,Cand D with BC=CD . The time taken by the particle to move from B to C is 2 seconds and from C to D is 1 second. Find the time taken to move from Ato B ?

Sol : Let AB=y:BC=CD=h and t_{AB}=t



then $y = \frac{1}{2}gt^2$

$y + h = \frac{1}{2}g\left(t+2\right)^2$

and $y + 2h = \frac{1}{2}g\left(t+3\right)^2$

solving these three equations , we get t=0.5 s

Problem : 82

A ball is thrown vertically upward with a velocity 'u' from the balloon descending with velocity v. After what time, the ball will pass by the balloon ?

Sol : $S_r = u_r t + \frac{1}{2}a_r t^2$

$O = \left(v+u\right) - \frac{1}{2}gt^2$

$t = \frac{2\left(v+u\right)}{g}$

Problem : 83

A ball dropped from the 9th storey of a multi - storeyed building reaches the ground in 3 second. In the first second of its free fall, it passes through n storeys, where n is equal to (Take g = 10 m s^{-2})

Sol : $9y = \frac{1}{2}\times10\times3\times3$ or y = 5 m

Again , $n\times5 = \frac{1}{2}\times10\times1\times1 = 5$ or n = 1

Problem : 84

A stone is dropped into water from a bridge 44.1 m above the water. Another stone is thrown vertically downward 1 s later. Both strike the water simultaneously. What was the initial speed of the second stone ?

Sol: $t = \sqrt{\frac{2\times44.1}{9.8}}s = \sqrt{9}s = 3s,$

$44.1 = v\times2 + \frac{1}{2}\times9.8\times2\times2$

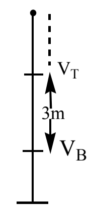
or $2v = 44.1 - 4.9\times4 = 24.5$

or $v = \frac{24.5}{2}ms^{-1} = 12.25\ ms^{-1}$

Problem : 85

A ball is dropped from the top of a building. It takes 0.5s to fall past the 3m length of a window some distance from the top of the building. If the velocity of the ball at the top and at the bottom of the window are V_T and V_B respectively then $V_T + V_B$ = ?

Sol. $S = \left(\frac{u+v}{2}\right)t$



$3 = \left(\frac{v_T + v_B}{2}\right)0.5$

$\boxed{V_T + V_B = 12m/s}$

Problem : 86

Two balls are projected vertically upwards with velocitees u_1 and u_2 from the ground with a time gap of n seconds. Find the time after which they meet

Sol. If they meet at a height h then

$h = u_1 t - \frac{1}{2}gt^2 = u_2\left(t-n\right) - \frac{1}{2}g\left(t-n\right)^2$

$u_1 t - \frac{1}{2}gt^2 = u_2 t - u_2 n - \frac{1}{2}\not{g}t^2 - \frac{1}{2}gn^2 + gtn$

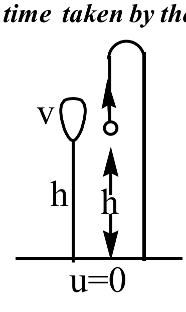
$\boxed{t = \frac{u_2 n + \frac{1}{2}gn^2}{\Delta u + gn}}$

b) $\boxed{t = \frac{u}{g} + \frac{n}{2}}$ if $u_1 = u_2$

Problem : 87

A balloon starts from rest from the ground and moves with uniform acceleration g/8. When it reaches a height h a ball is dropped from it the time taken by the ball to reach the ground is

Sol : $v = \sqrt{\frac{2gh}{8}} = \frac{\sqrt{gh}}{2}$



$-h = vt - \frac{1}{2}gt^2$

$-h = \frac{\sqrt{gh}}{2}t - \frac{1}{2}gt^2$

$\frac{1}{2}gt^2 - \frac{\sqrt{gh}}{2}t - h = 0$

Simplifing and taking only the positive value as negative value of t is not acceptable we get

$$t = 2\sqrt{\frac{h}{g}}$$

Problem : 88

A boy sees a ball go up and then down through a window 2.45m high. If the total time that ball is in sight in 1s, the height above the window the ball rises is approximately

Sol : Time during upward crossing of 2.45m

= time during downwad crossing = $\frac{1}{2}s$

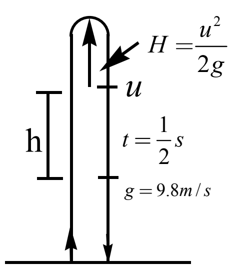
$$h = ut + \frac{1}{2}gt^2$$

$$2.45 = u.\frac{1}{2} + \frac{1}{2}\times 9.8\times\frac{1}{4}$$

u = 4.9 – 2.45 = 2.45

$$H = \frac{u^2}{2g} = \frac{2.45\times 2.45}{2\times 9.8}$$

$$H \simeq 0.3m$$



4.29 PROJECTILE

The horizontal component of v elocity remains constant all along . (since acceleration due to gravity has no component along the horizontal.

Note 14: The vertical component of velocity,

a) Goes on decreasing during the ascent

b) goes on increasing during the descent

c) becomes zero at the highest point

Note 15: Thus velocity of a projectile is maximum at projection point (equal to u)

Note 16: velocity of the projectile is minimum at highest point (equal to $u\cos\theta$)

Note 17: change in the velocity of the projectile is equal to $2u\sin\theta$

$$\left(\begin{array}{l}\Delta v_y = v_f - v_i = u\sin\theta - (-u\sin\theta)\\ = 2u\sin\theta \; and \; \Delta v_x = 0\end{array}\right)$$

Note 18:

similarly change in momentum = $2mu\sin\theta$

Note 19: average velocity of the projectile during the entire journey =

$$\frac{total\ displacement}{total\ time} = \frac{range}{time\ of\ flight}$$

$$= \frac{(u\cos\theta)\times T}{T} = u\cos\theta$$

Note 20 : angle between velocity and accelaration of a projectile,

a) is between 90^0 to 180^0 during the ascent i.e the dot product of velocity and acceleration is -ve during the ascent

b) is between 0^0 to 90^0 during the descent i.e the dot product of velocity and acceleration is +ve during the descent

c) is 90^0 at the highest point i.e the dot product of velocity and acceleration is 0 at the highest point

Note 21: At the projection point ,

$$total\ energy = E_{to} = \frac{1}{2}mu^2 (i.e, it\ is\ purely\ kinetic)$$

Note 22 : At the highest point of the projectile ,

a) kinetic energy $E_k = \frac{1}{2}mu^2\cos^2\theta = E_{to}\cos^2\theta$

b) potential energy $E_p = \frac{1}{2}mu^2\sin^2\theta = E_{to}\sin^2\theta$

c) ratio of potential to kinetic energies

$$= \frac{E_p}{E_k} = \tan^2\theta$$

Note 23: If 'T' is the time of flight of a projectile, maximum height $H = \frac{1}{8}gT^2$

Sol : We know that $T = \frac{2u\sin\theta}{g} \Rightarrow u = \frac{gT}{2\sin\theta}$

$$\therefore H = \frac{u^2\sin^2\theta}{2g} = \left(\frac{gT}{2\sin\theta}\right)^2 \times \frac{\sin^2\theta}{2g}$$

$$= \frac{g^2T^2}{4\sin^2\theta}\times\frac{\sin^2\theta}{2g} = \frac{1}{8}gT^2$$

Note 24: For a projectile, angle of projection [θ], range [R] and maximum height [H] are **related as** $\left(Tan\theta = \frac{4H}{R}\right)$

Sol : Maximum height $H = \frac{u^2\sin^2\theta}{2g}$ .....(1)

Range $R = \frac{u^2 \sin 2\theta}{g}$ .....(2)

$\frac{(1)}{(2)} \Rightarrow \frac{u^2 \sin^2\theta}{2g} \times \frac{g}{u^2 \sin^2\theta}$

$= \frac{u^2 \sin^2\theta}{2g} \times \frac{g}{u^2 2\sin\theta\cos\theta}$

$\Rightarrow \frac{Tan\theta}{4} = \frac{H}{R} \Rightarrow Tan\theta = \frac{4H}{R}$

Note 25: The time of flight (T), Range (R), and angle of projection (θ) are related as,

$(gT^2 = 2R\ Tan\theta)$

Sol : $T = \frac{2u\sin\theta}{g}$ ....[1]

and $R = \frac{u^2 \sin 2\theta}{g}$ ....[2]

$\frac{(1)^2}{(2)} \Rightarrow \frac{T^2}{R} = \frac{4u^2\sin^2\theta}{g^2} \times \frac{g}{u^2 \sin 2\theta}$

$= \frac{4u^2\sin^2\theta}{g^2} \times \frac{g}{u^2(2\sin\theta\cos\theta)}$

$\Rightarrow \frac{2Tan\theta}{g} = \frac{T^2}{R}$

$\Rightarrow gT^2 = 2R\ Tan\theta$

Note 26: $\boxed{R\tan\theta = 4H = \frac{1}{2}gT^2}$

Note 27: range of a projectle is maximum when angle of projection = 45^0 ($\because R = \frac{u^2\sin 2\theta}{g}$, R is maximum if $\sin 2\theta$ is maximum i.e if $2\theta = 90^0\ or\ \theta = 45^0$

Note 28: range of a projectile = maximum height if $\theta = \tan^{-1} 4\ or\ 76^0$

Sol : we know that $\tan\theta = \frac{4H}{R}$ from this

$R = H \Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1} 4 = 76^0$

Note 29: For projectile, in the case of complimentary angles,

a) Ranges are same

Sol : If $\theta\ and\ (90-\theta)$ are angles of projection, we have

$R_1 = \frac{u^2\sin 2\theta}{g}\ and\ R_2 = \frac{u^2 \sin 2(90-\theta)}{g}$

$\Rightarrow R_1 = \frac{u^2\sin 2\theta}{g}\ and\ R_2 = \frac{u^2\sin(180-2\theta)}{g}$

$\Rightarrow R_1 = R_2$

b) If H_1, H_2 are maximum heights, $H_1 + H_2 = \frac{u^2}{2g}$

Sol:

we have $H_1 = \frac{u^2\sin^2\theta}{2g}$, $H_2 = \frac{u^2\sin^2(90-\theta)}{2g}$

$H_1 + H_2 = \frac{u^2\sin^2\theta}{2g} + \frac{u^2\cos^2\theta}{2g}$

$\therefore H_1 + H_2 = \frac{u^2}{2g}$

c) $R_1 = R_2 = \boxed{R = 4\sqrt{H_1 H_2}}$

d) If T_1, T_2 are times of flight, $T_1 T_2 = \frac{2R}{g}$

Sol: We have

$T_1 = \frac{2u\sin\theta}{g}\ and\ T_2 = \frac{2u\sin(90-\theta)}{g}$

$\Rightarrow T_1 T_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$

$= \frac{2u^2\sin 2\theta}{g^2}$

$T_1 T_2 = \frac{2R}{g}$ i $\therefore \boxed{R = \frac{1}{2}gT_1T_2}$

Note 30: If horizontal and vertical displacements of a projectile are respectively x = at and y=bt-ct², then velocity of projection $u = \sqrt{a^2 + b^2}$ and angle of projection

$\theta = \tan^{-1}\frac{b}{a}$

Note 31: For a projectile,

'y' component of velocity at half of maximum height $=\dfrac{u\sin\theta}{\sqrt{2}}$

Proof: Applying $v^2 - u^2 = 2as$ for upward journey of a projectile,

we have, $u = u\sin\theta$, a= -g,

$$S=\frac{1}{2}H=\frac{1}{2}\frac{u^2\sin^2\theta}{2g}$$

Substituting these values we get

$$v^2-\left(u\sin\theta\right)^2=-2g\times\frac{1}{2}\times\frac{u^2\sin^2\theta}{2g}$$

$$\therefore v^2=u^2\sin^2\theta-\frac{u^2\sin^2\theta}{2}=\frac{u^2\sin^2\theta}{2}$$

$$\Rightarrow v=\frac{u\sin\theta}{\sqrt{2}}$$

Note 32: velocity of a projectile at half of maximum height $= u\sqrt{\dfrac{1+\cos^2\theta}{2}}$

Proof : Velocity at any instant is $v=\sqrt{v_x^2+v_y^2}$

But $v_x = u_x = u\cos\theta$ at any point

while $v_y=\dfrac{u\sin\theta}{\sqrt{2}}$ (at half of maximum height)

$$\therefore v=\sqrt{\left(u\cos\theta\right)^2+\left(\frac{u\sin\theta}{\sqrt{2}}\right)^2}$$

Simplifying we get

$$v=u\sqrt{\frac{1+\cos^2\theta}{2}}$$

Note 33: The physical quantities which remains constant during projectile motion are

i) acceleration due to gravity g

ii) total energy $E_0=\dfrac{1}{2}mu^2$

iii) horizontal component of the velocity $u\cos\theta$

Note 34: If air resistance is taken into consideration then

a) trajectory departs from parabola

b) time of filght may increase or decrease

c) the velocity with which the body strikes the ground decreases

d) maximum height decreases

e) striking angle increases

f) range decreases

Note 35: A projectile is fired with a speed u at an angle θ with the horizontal. Its speed when its direction of motion makes an angle α with the horizontal

$v = u\cos\theta \sec.\alpha$

Explanation : Horizontal component of velocity remains constant

$\therefore v\cos\alpha = u\cos\theta$ $\qquad v = u\cos\theta\sec.\alpha$

Note 36 : A body is dropped from a tower. If wind exerts a constant horizontal force the path of the body is a straight line

Note 37 : The path of projectile as seen from another projectile:

$$x_1=u_1\cos\theta_1 t \qquad y_1=u_1\sin\theta_1 t-\frac{1}{2}gt^2$$

$$x_2=u_2\cos\theta_2 t \qquad y_2=u_2\sin\theta\, t-\frac{1}{2}gt^2$$

$$\Delta x=\left(u_1\cos\theta_1-u_2\cos\theta_2\right)t$$

$$\Delta y=\left(u_1\cos\theta_1-u_2\cos\theta_2\right)t$$

$$\frac{\Delta y}{\Delta x}=\frac{u_1\sin\theta_1-u_2\sin\theta_2}{u_1\cos\theta_1-u_2\cos\theta_2}$$

i) If $u_1\sin\theta_1=u_2\sin\theta_2$

i.e., initial vertical velocities are equal slope $\dfrac{\Delta y}{\Delta x}=0$

$\Rightarrow$ the path is horizontal straight line

ii) If $u_1\cos\theta_1=u_2\cos\theta_2$

i.e., initial horizontal velocities are equal

slope $\dfrac{\Delta y}{\Delta x}=\infty$

$\Rightarrow$ the path is a vertical straight line

iii) $u_1\sin\theta_1>u_2\sin\theta_2$

$u_1\cos\theta_1>u_2\cos\theta_2$

$\Rightarrow$ the path is a straight line with +ve slope

iv) $u_1\sin\theta_1>u_2\sin\theta_2;\ u_1\cos\theta_1<u_2\cos\theta_2$

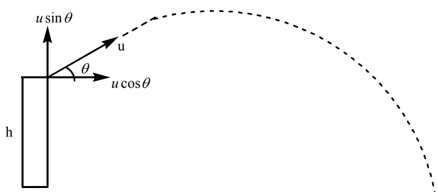
or $u_1\sin\theta_1<u_2\sin\theta;\ \ u_2\cos\theta_1<u_2\cos\theta_2$

$\Rightarrow$ the path is straight line with –ve slope

Note 38 : Two bodies thrown with same speed from the same point at the same instants but at different angles can rever collide in air

$\because$ (x = $u\cos\theta t$, , y = $u\sin\theta - \frac{1}{2}gt^2$, x, y coordinates always differ)

Note 39 : A body is projected up with a velocity u at an angle θ to the horizontal from a tower of height h as shown. It is clear that such a body also traces a parabolic path. The time taken to reach ground is arrived as explained below .

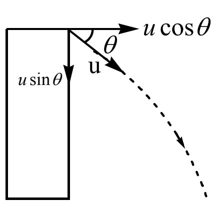


The components of velocity are as shown.
Here the body can be treated as a body projected vertically up with a velocity $u\sin\theta$ from a tower of height h . Hence the equation of motion on reaching the foot of the tower is

$$h = -(u\sin\theta)t + \frac{1}{2}gt^2$$

by using the formula for height of tower

Note 40 : A body is projected down with a velocity u at an angle θ to the horizontal from a tower of height h as shown. It is clear that such a body traces a parabolic path. The time taken to reach ground is arrived as explained below.



The components of velocity are as shown. Here the body can be treated as a body projected vertically down with a velocity $u\sin\theta$ from a tower of height h. Hence the equation of motion on reaching the foot of the tower is

$$h = (u\sin\theta)t + \frac{1}{2}gt^2$$

(using $S = u^1 t + \frac{1}{2}gt^2$ where $u^1 = u\sin\theta$)

Problem : 89

A stone is to be thrown so as to cover a horizontal distance of 3 m. If the velocity of the projectile is 7 ms^{-1}, find

(i) the angle at which it must be thrown,

(ii) the largest horizontal displacement that is possible with the projection speed of 7 ms^{-1}.

Sol. (a) Given that, u=7ms^{-1}, R = 3m

$R = \frac{u^2\sin 2\theta}{g}$ $\quad \Rightarrow 3 = \frac{(7)^2\sin 2\theta}{9.8}$

$\Rightarrow \sin 2\theta = 3/5$

$\Rightarrow 2\theta = 37^o$ or $180^o - 2\theta$

$\Rightarrow \theta = 18^o\ 30'$ or $\theta = 71^o\ 30'$

Hence a range of 3m is possible with two angles of projections.

(b) For maximum range with a given velocity, the angle of projection, $\theta = 45^o$

$R_{max} = \frac{(7)^2\sin 2(45^0)}{9.8} = 5m$

Problem : 90

The speed with which a bullet can be fired is 150 ms^{-1}. Calculate the greatest distance to which it can be projected and also the maximum height to which it would rise.

Hint : $R = \frac{u^2\sin 2\alpha}{g}, H_{max} = \frac{u^2\sin^2\alpha}{2g}$ Here, $\alpha = 45^0$

Ans. 2295.14m, 573.97m

Problem : 91

A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial velocity 240 m/s reach the target in the absence of air drag?

Sol. Here, v_0 = 240 ms^{-1}, R = 5.10 km = 5100 m,

g = 9.8 ms^{-2}, α = ?

$$R = \frac{v_0^2\sin 2\alpha}{g}$$

$$\sin 2\alpha = \frac{Rg}{v_0^2}$$

$\Rightarrow \alpha = 30°$ or $60°$

using = $T = \dfrac{2v_0 \sin\alpha}{g}$

When, $\alpha = 30°,\ T_1 = \dfrac{2 \times 240 \times 0.5}{9.8} = 24.5$ s

When, $\alpha = 60°, T_2 = \dfrac{2 \times 240 \times 0.867}{9.8} = 42.41$ s

Problem : 92

The horizontal range of a projectile is $2\sqrt{3}$ times its maximum height. Find the angle of projection.

Sol. Hint $R\tan\theta = 4H$;

Ans : $\theta = \tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

Problem : 93

The ceiling of a long hall is 20 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall ($g = 10\ ms^{-2}$) ?

Sol. Here, H = 20 m, u = 40 ms^{-1}.

Suppose the ball is thrown at an angle θ with the horizontal.

Now, $H = \dfrac{u^2 \sin^2\theta}{2g}$ $\Rightarrow 20 = \dfrac{(40)^2 \sin^2\theta}{2 \times 10}$

or, $\sin\theta = 0.5$ or, $\theta = 30°$

Now $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{(40)^2 \times \sin 120°}{10}$

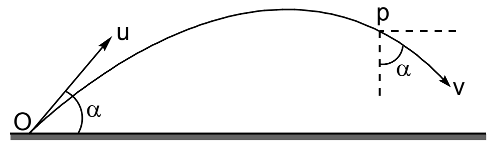
$= \dfrac{(40)^2 \times 0.866}{10} = 138.56cm$

Problem : 94

If at point of projection, the velocity of a particle is "u" and is directed at an angle "α" to the horizontal, then show that it will be moving at right angles to its initial direction after a time $\dfrac{(u \operatorname{cosec} \alpha)}{g}$

Sol. Let "t" be the time after which velocity becomes perpendicular to its initial direction.

As u and v are perpendicular, the angle between v and vertical will be a.



Initial velocity 'u' $= (u\cos\alpha\ \hat{i} + u\sin\alpha\ \hat{j})$

After *t* sec, velocity 'v' $= \left\{u\cos\alpha\ \hat{i} + (u\sin\alpha - gt)\hat{j}\right\}$

$\because$ These are perpendicular their dot product is zero.

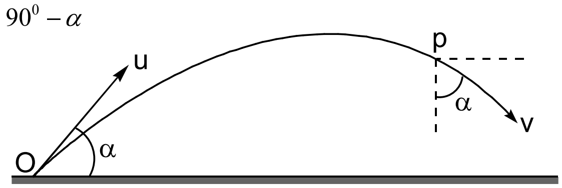
$\therefore (u\cos\alpha\, i + u\sin j)\cdot\left\{u\cos\alpha\, i + (u\sin\alpha - gt)j\right\} = 0$

and $t = \dfrac{u \operatorname{cosec}\alpha}{g}$

Problem : 95

Find the velocity in the above problem at the instant when the instantaneous velocity is perpendicular to velocity of projection

Sol. From the figure its clear that angle made by the instantaneous velocity vector with horizontal is $90^0 - \alpha$



since the horizontal component of velocity does not change, we have,

$v\cos\left(90^0 - \alpha\right) = u\cos\alpha \Rightarrow v\sin\alpha = u\cos\alpha$

$\Rightarrow v = u\cot\alpha$

Problem : 96

A particle is projected from the origin in X–Y plane. Acceleration of particle in Y direction is α. If equation of path of the particle is $y = ax - bx^2$, then find initial velocity of the particle.

Sol. $y = ax - bx^2$

$y = x\tan\theta - \dfrac{\alpha x^2}{2u^2\cos^2\theta}$

$\tan\theta = a$ and $\dfrac{\alpha}{2u^2\cos^2\theta} = b$

$u = \sqrt{\dfrac{\alpha\left(1+a^2\right)}{2b}}$.

Problem : 97

A particle is projected from the origin. If $y = ax - bx^2$, is the equation of the trajectory then find

i) angle of projection ii) range

iii) maximum height

Sol. comparing the given equation with the equation of the trajectory of a projectile given by

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

i) we have $\tan\theta = a \Rightarrow$ angle of projection $\theta = \tan^{-1} a$

ii) For y = 0 to the x coordinat gives range

$\therefore R = \frac{a}{b}$

iii) $\tan\theta = \frac{4H}{R}$ $\therefore H = \frac{a^2}{4b}$

Problem : 98

If $y = x - \frac{1}{2}x^2$ is the equation of a trajectory, find the time of flight.

Sol. We have $y = x - \frac{1}{2}x^2 = x(1 - x/2)$

If y = 0, then either x = 0 or x = 2.

Hence the range of the motion is 2.

For half the range, x = 1, then y = 1/2

Hence maximum height attained H = 1/2.

Time to reach maximum height,

$$t_a = \sqrt{\frac{2H}{g}} = \sqrt{\frac{1}{g}}$$

Time of flight, $T = 2t = 2\sqrt{\frac{1}{g}}$

***Problem : 99**

A ball is thrown with velocity $10ms^{-1}$ at an angle of $\alpha = 45°$ to the horizontal. Find i) the height of which the ball will rise to ii) the distance x from the point of projection to the point where it reaches to the ground and iii) the time during with the ball will be in motion (neglect the air resistance) ($g = 10ms^{-2}$)

Hint : i) $H_{max} = \frac{u^2\sin^2\theta}{2g}$

ii) $R = \frac{u^2\sin 2\theta}{g}$ iii) $T = \frac{2u\sin\theta}{g}$

Ans : i) 2.1m ii) 10m iii) 1.414s

Problem : 100

A body is projected with velocity u at an angle of projection θ with the horizontal. The body makes 30^0 with the horizontal at t = 2 second and then after 1 second it reaches the maximum height. Then find (a) angle of projection (b) speed of projection

Sol : During the projectile motion, angle at any instant t is suchthat

$$\tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

For t = 2seconds, $\alpha = 30^0$

$$\frac{1}{\sqrt{3}} = \frac{u\sin\theta - 2g}{u\cos\theta} \quad --(1)$$

For t = 3 seconds, at the highest point $\alpha = 0^0$

$$\therefore 0 = \frac{u\sin\theta - 3g}{u\cos\theta}$$

$\therefore 3 = \frac{u\sin\theta}{g}$ or $u\sin\theta = 3g$ _______(2)

using eq. (1) and eq.(2)

$u\cos\theta = \sqrt{3}g$ _______(3)

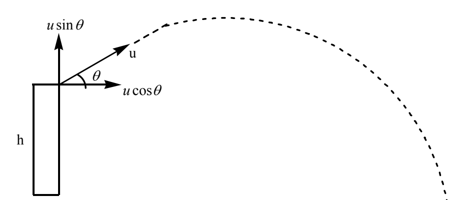
Eq.(2) $\div$ eq.(3) give $\theta = 60^0$ squaring and adding equation (2) and (3)

$u = 20\sqrt{3}$ m/s.

Problem : 101

A ball is thrown from the top of a tower of 61 m high with a velocity $24.4\ ms^{-1}$ at an eleva- tion of 30° above the horizontal. What is the distance from the foot of the tower to the point where the ball hits the ground ?

Sol : (Hint) :



$h = \frac{1}{2}gt^2 - (u\sin\theta)t$ $\Rightarrow t = 5\,seconds$

Also, $d = (u\cos\theta)t = 105.65$ m

Problem : 102

A particle is projected at an angle of elevation α and after t secons it appears to have an elevation β as seen from the point of projection. Find the initial velocity of projection.

Sol. $\tan\beta = \frac{y}{x} = \frac{u\sin\alpha\, t - \frac{1}{2}gt^2}{u\cos\alpha\, t}$

$$u(\sin\alpha - \cos\alpha\tan\beta) = \frac{gt}{2}$$

$$u = \frac{gt}{2(\sin\alpha - \cos\alpha\tan\beta)}$$

$$u = \frac{gt\cos\beta}{2\sin(\alpha-\beta)}$$

Problem : 103

A rifle with a muzzle velocity of 100 ms^{-1} shots a bullet at a small target 30 m away in the same horizontal line. How much height above the target must the rifle be aimed so that the bullet will hit the target?

Hint : $R = \frac{u^2\sin 2\theta}{g}$ **Ans.** $\theta = 0.015$

Problem : 104

A hunter aims his gun and fires a bullet directly at a monkey on a tree. At the instant the bullet leaves the barrel of the gun, the monkey drops. Will the bullet hit the monkey ?

Sol. Horizontal distance travelled

$OB = x = u\cos\theta\, t$ or $t = \frac{x}{u\cos\theta}$

For motion of bullet from O to B, the vertical height $AB = u\sin\theta\, t - \frac{1}{2}gt^2$

$= u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}gt^2$

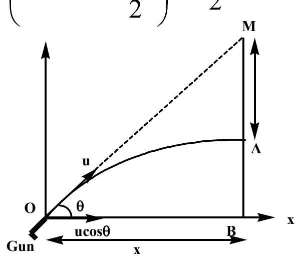
$= x\tan\theta - \frac{gt^2}{2}$ ......(i)

Also from figure $MB = x\tan\theta$

Now the height through which monkey falls

$y = MA = MB - AB$

$= x\tan\theta - \left(x\tan\theta - \frac{gt^2}{2}\right) = \frac{1}{2}gt^2$



Thus, in time t the bullet passes through A a vertical distance $\frac{1}{2}gt^2$ below M.

The vertical distance through which the monkey fall in time t. $s = \frac{1}{2}gt^2$

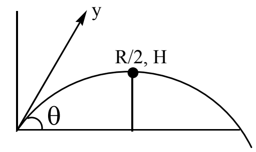
Thus, the bullet and the monkey will always reach at point A at the same time.

Problem : 105

A particle is projected from the ground with an initial speed u at an angle θ with horizontal. What is the average velocity of the particle between its point of projection and highest point of trajectory ?

Sol. $V_{av} = \frac{\text{Total displacement}}{\text{Total time}}$

$V_{av} = \frac{\sqrt{\frac{R^2}{2} + H^2}}{\frac{u\sin\theta}{g}}$



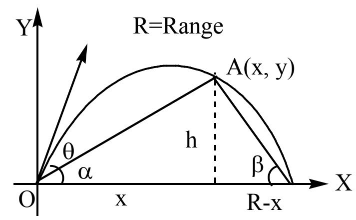
$V_{av} = \frac{\sqrt{R^2+4H^2}}{\frac{2u\sin\theta}{g}} = \frac{\sqrt{\left(\frac{u^2}{g}\sin 2\theta\right) + 4\left(\frac{u^2}{2g}\sin^2\theta\right)^2}}{\frac{2u\sin\theta}{g}}$

$= \frac{u^2}{g}\frac{\sqrt{4\sin^2\theta\cos^2\theta + \sin^4\theta}}{\frac{2u\sin\theta}{g}} = \frac{u}{2}\sqrt{1+3\cos^2\theta}$

Problem : 106

A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ be the angle of projection, prove that $\tan\theta = \tan\alpha + \tan\beta$.

Sol : The situation is shown in figure. From figure,we have



$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x}$

$\tan\alpha + \tan\beta = \frac{yR}{x(R-x)}$ ——— (1)

But equation of trajectory is $y = x\tan\theta\left[1 - \frac{x}{R}\right]$

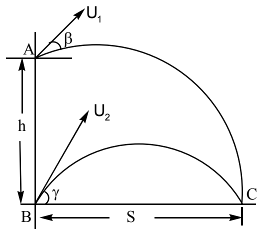
$\tan\theta = \frac{yR}{x(R-x)}$

From Eqs. (i) and (ii), $\tan\theta = \tan\alpha + \tan\beta$

Problem : 107

Two shots are fired simultaneously from the top and bottom of a vertical tower AB at angles β and γ with horizontal respectively. Both shots strikes at the same point C on the ground at distance 'S' from the foot of the tower at the same time. Show that the hight of the

tower is $S(\tan\gamma - \tan\beta)$.



Sol. $S = (u_1\cos\beta)t = (u_2\cos\gamma)t$

$\therefore t = \dfrac{S}{u_1\cos\beta} = \dfrac{S}{u_2\cos\gamma}$

Let, height of tower be h

$-h = (u_1\sin\beta)t - \dfrac{1}{2}gt^2 \qquad 0 = (u_2\sin\gamma)t - \dfrac{1}{2}gt^2$

or $(u_1\sin\beta)t + h = (u_2\sin\gamma)t$

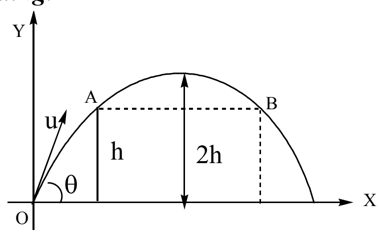
$\therefore u_1\sin\beta.\dfrac{S}{u_1\cos\beta} + h = \dfrac{u_2\sin\gamma}{u_2\cos\gamma}.S$

$h + S\tan\beta = S\tan\gamma$

$\therefore h = S(\tan\gamma - \tan\beta)$

Problem : 108

A stone is projected from the point of a ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height 2h above the ground. If at the instant of projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocities of the bird and the stone, if the stone still hits the bird while descending.



Sol. Let θ be the angle of projection and u be the velocity of projection. It is given that the maximum height of the projectile is 2 h, we have

$u\sin\theta = \sqrt{4gh}$

If time taken by the stone to reach points A and B are t_1 and t_2, then t_1 and t_2 are the roots of the equation

$h = u\sin\theta t - \dfrac{1}{2}gt^2$ or $gt^2 - 2ut\sin\theta + 2h = 0$

Solving, $t = \dfrac{u\sin\theta}{g} \pm \dfrac{\sqrt{u^2\sin^2\theta - 2gh}}{g}$

Using $\quad u\sin\theta = \sqrt{4gh}$

$t = \sqrt{\dfrac{4h}{g}} \pm \sqrt{\dfrac{2h}{g}}$

Thus, we have $t_1 = \sqrt{\dfrac{4h}{g}} - \sqrt{\dfrac{2h}{g}}$

and $t_2 = \sqrt{\dfrac{4h}{g}} + \sqrt{\dfrac{2h}{g}}$

Now, the distance AB can be written as

$vt_2 = u\cos\theta(t_2 - t_1)$

(v = velocity of the bird)

Ratio of horizontal velocities

$$\frac{v}{u\cos\theta} = \frac{t_2 - t_1}{t_2} = \frac{2}{\sqrt{2}+1}$$

Problem : 109

The velocity of a projectile when at its greatest height is $\sqrt{\dfrac{2}{5}}$ ***of its velocity when at half of its greatest height find the angle of projection***

Sol : Step 1 : we know that, velocity of a projectile at half of maximum height $= u\sqrt{\dfrac{1+\cos^2\theta}{2}}$

Step 2 : given that $u\cos\theta = \sqrt{\dfrac{2}{5}} \times u\sqrt{\dfrac{1+\cos^2\theta}{2}}$

Squaring on both sides

$u^2\cos^2\theta = \dfrac{2}{5}u^2\left(\dfrac{1+\cos^2\theta}{2}\right)$

$10\cos^2\theta = 2 + 2\cos^2\theta$

$\Rightarrow 8\cos^2\theta = 2 \Rightarrow \cos^2\theta = \dfrac{1}{4} \Rightarrow \theta = 60^0$

Problem : 110

Afoot ball is kicked of with an initial speed of 19.6 m/sec at a projection angle 45⁰. A receiver on the goal line 67.4 m away in the direction of the kick starts running to meet the ball at that instant. What must his speed be if he is to catch the ball before it hits the ground ?

Sol : $R = \dfrac{u^2\sin 2\theta}{g} = \dfrac{(19.6)^2 \times \sin 90}{9.8}$

or $R = 39.2$ metre.

Man must run 67.4 m - 39.2 m = 28.2 m

in the time taken by the ball to come to ground.

Time taken by the ball.

$$t=\frac{2u\sin\theta}{g}=\frac{2\times19.6\times\sin45^0}{9.8}=\frac{4}{\sqrt2}$$

$$t=2\sqrt2=2\times1.41=2.82\,\text{sec}.$$

Velocity of man $=\frac{28.2m}{2.82\,\text{sec}}=10\,m/\text{sec}$.

Problem : 111

A projectile has the maximum range of 500m. If the projectile is now thrown up on an inclined plane of 30⁰ with the same speed, what is the distance covered by it along the inclined plane ?

Sol : $R_{\max}=\frac{u^2}{g}$

$\therefore 500=\frac{\boldsymbol{u}^2}{\boldsymbol{g}}$ *or* $\boldsymbol{u}=\sqrt{500\boldsymbol{g}}$

$v^2-u^2=2gs$

0 - 500 g = 2 x (-g sin 30⁰) x $\boldsymbol{x}$

$\boldsymbol{x}$ = 500 m.

Problem : 112

Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\pi/3$ and its maximum height is y_1, then what is the maximum height of the other ?

Sol : Here speeds of projection and ranges are same and hence angles of projections are

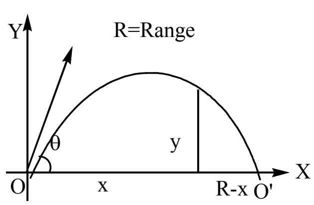
$\frac{\pi}{3}$ ***and*** $\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$

Now $\frac{\boldsymbol{y}_2}{\boldsymbol{y}_1}=\frac{\boldsymbol{u}^2\sin^2\theta_2}{2\boldsymbol{g}}\times\frac{2\boldsymbol{g}}{\boldsymbol{u}^2\sin^2\theta_1}$

$=\frac{\sin^2\theta_2}{\sin^2\theta_1}=\frac{\sin^2(\pi/6)}{\sin^2(\pi/3)}=\frac{1}{3}$ $\boldsymbol{y}_2=\frac{\boldsymbol{y}_1}{3}$

Problem : 113

A body projected from a point 'O' at an angle θ, just crosses a wall 'y' m high at a distance 'x' m from the point of projection and strikes the ground at O' beyond the wall as shown, then find height of the wall ?



Ans : $y=x\tan\theta\left[1-\frac{x}{R}\right]$

Sol. We know that the equation of the trajectory is

$y=x\tan\theta-\frac{gx^2}{2u^2\cos^2\theta}$ can be written as

$$y=x\tan\theta-\left(\frac{gx^2}{2u^2\cos^2\theta}\right)\frac{\sin\theta}{\sin\theta}$$

$$y=x\tan\theta-\frac{gx^2\tan\theta}{u^2(2\sin\theta\cos\theta)}$$

$$\Rightarrow y=x\tan\theta-\frac{x^2\tan\theta}{u^2\sin2\theta/g}$$

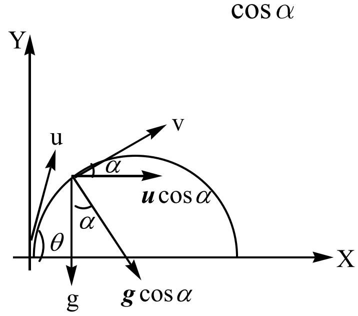
$\Rightarrow y=x\tan\theta\left[1-\frac{x}{R}\right]$ [since $R=\frac{u^2\sin2\theta}{g}$]

Problem : 114

A body is projected with a velocity 'u' at θ to the horizontal. Find radius of curvature of the trajectory when the velocity vector makes α with horizontal.

Sol: Let v be the velocity of particle when it makes α with horizontal . Then

$v\cos\alpha=u\cos\theta$ or $v=\frac{u\cos\theta}{\cos\alpha}$



it is clear that $g\cos\alpha$ plays the role of radial acceleration

$$g\cos\alpha=\frac{v^2}{R}\Rightarrow R=\frac{v^2}{g\cos\alpha}$$

$$=\left(\frac{u\cos\theta}{\cos\alpha}\right)^2\left(\frac{1}{g\cos\alpha}\right)=\frac{u^2\cos^2\theta}{g\cos^2\alpha}$$

Note : When the projectile is at the highest point, its clear that $\alpha=0^0$.

$$R=\frac{\boldsymbol{u}^2\cos^2\theta}{\boldsymbol{g}}$$

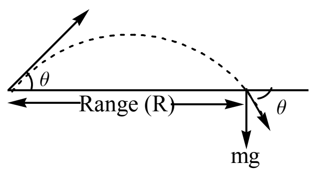
Problem : 115

For a projectile, projected with a velocity u at an angle θ to the horizontal. Find the magnitude of torque about the origin when it strikes the ground

Sol. we know that torque

τ = force x perpendicular distance from the origin on to the line of action of force

$$= \text{mgxrange} = \frac{mg \times u^2 \sin 2\theta}{g}$$



Problem : 116

A grass hopper can jump maximum distance of 1.6m. It spends negligible time on the ground. How far can it go in 10 seconds?

$\frac{u^2}{g} = 1.6 \quad u^2 = 16 \quad u = 4m/s$

$4\cos\theta = 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2} m/s$

$S = 4\cos\theta . t = 2\sqrt{2} \times 10$

$S = 20\sqrt{2} m$

Problem : 117

A particle is projected with a velocity of $10\sqrt{2}$ m/s at an angle of 45^0 with the horizontal. Find the interval between the moments when speed is $\sqrt{125}\, m/s$.

Sol. (g = 10m/s^2)

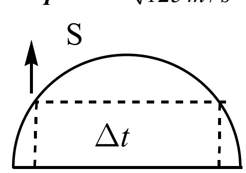
$u_n = 10, u_y = 0$

$v^2 = v_x^2 + v_y^2$

$125 = 100 + v_y^2$

$v_y = 5$

$\Delta t = \frac{2v}{g} = \frac{2 \times 5}{10} = \boxed{1s}$



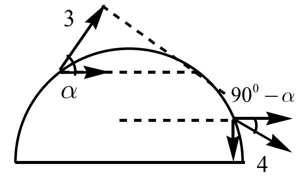
Problem : 118

A projective of 2kg was velocities 3m/s and 4m/s at two points during its flight in the uniform gravitational field of the earth. If these two velocities are $\perp$ to each other then the minimum KE of the particle during its flight is

$V_1 \cos\alpha = V_2 \cos(90 - \alpha)$

$3\cos\alpha = 4\sin\alpha$

$\tan\alpha = \frac{3}{4}$



$\text{KE}_{\min} = \frac{1}{2} m v_1^2 \cos^2\alpha$

$= \frac{1}{2} \times 2 \times 3 \times \left(\frac{4}{5}\right)^2$

$= \frac{9 \times 16}{25} = \boxed{5.76J}$

Note 41 : If $\vec{u} = x\vec{i} + y\vec{j}$

$\vec{i}$ along horizontal $\vec{j}$ along vertical

$H = \frac{y^2}{2g},\ T = \frac{2y}{g},\ R = \frac{2xy}{g}$

Note 42 : $\vec{u} = a\vec{i} + b\vec{j} + c\vec{k}$

$\vec{i}$ –east $\vec{j}$ - north $\vec{k}$ -vertical

$u_x = \sqrt{a^2 + b^2}$ $u_y = c$

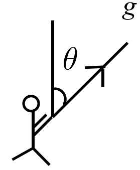
$T = \frac{2c}{g}; H = \frac{c^2}{2g},$

$R = \frac{2\left(\sqrt{a^2+b^2}\right)c}{g}$

Problem : 119

Wind imparts a horizontal acceleration of 0.4m/s^2 towards left. $\theta = ?$ for the ball to fall in the hand of thrower

Sol. $R = \frac{2u^2 \sin\theta\cos\theta}{g} = u_x T$



$T = \frac{2u\cos\theta}{g}$

$R^| = u_x T - \frac{1}{2} a T^2$

$O = \frac{2u^2 \sin\theta\cos\theta}{g} - \frac{1}{2} 0.4. T^2$

$\frac{2u^2 \sin\theta\cos\theta}{g} = \frac{1}{2} \times \frac{4}{10} \times \left(\frac{2 \times 2u\cos\theta}{g}\right)^2$

$Tan\ \theta = 0.4 \Rightarrow \boxed{\theta = \tan^{-1}(0.4)}$

Problem : 120

In the absence of wind the range and maximum height of a projectile were R and H. If wind imparts a horizontal acceleration a = g/4 to the projectile then find the maximum range and maximum height.

Sol : $H^1 = H\left(\because u\sin\theta \text{ remains same}\right)$

$T^| = T$

$R^| = u_x T + \frac{1}{2}aT^2$

$= R + \frac{1}{2}\,\frac{g}{4}T^2$

$= R + \frac{1}{8}\,gT^2$

$= R + H$

$\boxed{R^1 = R + H}$ $\boxed{H^1 = H}$

Problem : 121

$\vec{u} = 4\vec{i} + 4\vec{j}$. ***mass = 2kg. A constant force*** $\vec{F} = -20\vec{j}N$ ***acts an the body. Initially the body was at (0,0). Find the x coordinate of the point where its y coordinate is again zero.***

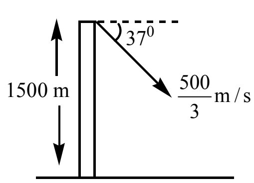
Sol : $a = \frac{20}{2} = 10$

$R = \frac{2u_x u_y}{10} = \frac{2\times 4\times 4}{10}$

$\boxed{R = 3.2m}$

Problem :.122

A particle is projected from a tower as shown in figure, then find the distance from the foot of the tower where it will stike the ground. (g = 10 m/s²)



Sol. $s = ut + \frac{1}{2}at^2$

$1500 = \frac{500}{3}\sin 37 + \frac{1}{2}10\times t^2$

$1500 = \frac{500}{3}\times\frac{3}{5}t + 5t^2$

$300 = 20t + t^2$

On solving , t = 10 s

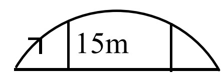
$\therefore$ horizontal distance = $u\cos\theta.\ T$

$= \frac{500}{3}\times\frac{4}{5}\times 10$

$= \frac{4000}{3}m$

Problem :123

A golfer standing on the ground hits a ball with a velocity of 52 m/s at an angle θ ***above the horizontal if*** $\tan\theta = \frac{5}{12}$ ***find the time for which the ball is atleast 15m above the ground?*** $\left(g = 10\ m/s^2\right)$



Sol. $v_y = \sqrt{u_y^{\,2} - 2gy}$

$= \sqrt{52\times 52\times\frac{5\times 5}{13\times 13} - 2\times 10\times 15}$

$= \sqrt{16\times 25 - 300} = 10$

$\Delta t = \frac{2u_y}{10} = \frac{2\times 10}{10} = \boxed{2s}$

Note 43:

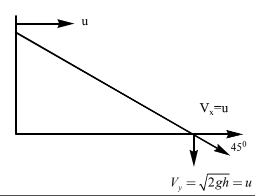
For an easier understanding, we consider that, [Motion of Horizontal projectile = Motion in y-direction like a freely falling body + Motion in x-direction with constant velocity.]

Application 9 :

If a body projected horizontally with velocity u from the top of a tower strikes the ground at an angle of 45⁰

$V_y = V_x$ gt=u

$\therefore t = \frac{u}{g}$

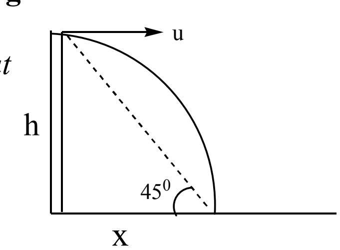


App 10

A body is projected horizontally from the top of a tower. The line joining the point of projection and the striking point make an angle of 45⁰ with the ground. Then h=u

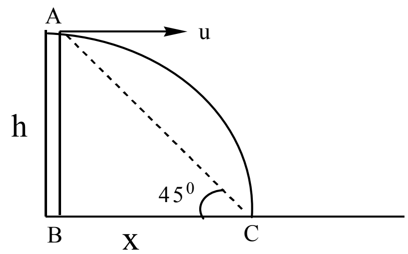
$\frac{1}{2}gt^2 = ut$

$t = \frac{2u}{g}$



Application 11:

A body is projected horizontally from the top of a tower . The line joining the point of projection and the striking point make an angle of 45^0 with the ground . Then, the displacement = $\sqrt{2}h \text{ or } \sqrt{2}X$



From the figure $\tan 45^0 = 1 = \frac{h}{X} \Rightarrow h = X$

$\therefore$ displacement AC $= \sqrt{(AB)^2 + (BC)^2}$

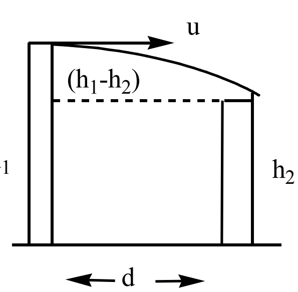
$= \sqrt{h^2 + H^2} = \sqrt{2}h \text{ or } \sqrt{2}X (\text{since } h = X)$

App 12 :

Two towers having heights h_1 and h_2 are separated by a distance 'd'. A person throws a ball horizontally with a velocity u from the top of the

1st tower to the top of the 2nd tower, then Time taken,

$$t = \sqrt{\frac{2(h_1 - h_2)}{g}}$$



Distance between the towers

$$d = ut = u\sqrt{\frac{2(h_1 - h_2)}{g}}$$

Application 13

An aeroplane flies horizontally with a velocity 'u' . If a bomb is dropped by the pilot when the plane is at a height 'h' then

a) the path of such as body is a vertical straight line as seen by the pilot and

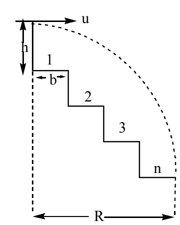
b) The path is a parabola as seen by an observer on the ground

c) the body will strike the ground at a certain horizontal distance. This distance is equal to the range given by $x = ut = u\sqrt{\frac{2u}{g}}$

Application 14:A ball rolls off the topof a staircase with a horizontal velocity u. If each step has height h and width b, then the ball will just hit the nth step if n equals to

Sol : $\therefore$ nb = ut and $nh = \frac{1}{2}gt^2$

$$\Rightarrow n = \frac{2hu^2}{gb^2}$$



App15 :

From the top of a towerone stone is thrown towards east with velocity u_1 and another is thrown towards north with velocity u_2. The distance between then after stihing the ground. $d = t\sqrt{u_1^2 + u_2^2}$

Application 16: Two bodies are thrown horizontally with velocities u_1, u_2 in mutually opposite directions from the same height. Then

a) time after which velocity vectors are perpendicular is $t = \frac{\sqrt{u_1 u_2}}{g}$.

For velocity vectors to be perpendicular after a time t, their dot product must be zero.

$$\therefore \ \bar{v}_1 . \bar{v}_2 = 0$$

$$(u_1\hat{i} - gt\hat{j}).(-u_2\hat{i} - gt\hat{j}) = 0$$

$$\therefore t = \frac{\sqrt{u_1 u_2}}{g}$$

b) Separation between them when velocity vectors are perpendicular is

$$X=(u_1+u_2)t=\frac{(u_1+u_2)\sqrt{u_1u_2}}{g}$$

c) Time after which their displacement vectors are perpendicular is $t=\frac{2\sqrt{u_1u_2}}{g}$

For displacement vectors to be perpendicular then their dot product must be zero

$$\left(u_1t\hat{i}-\frac{1}{2}gt^2\hat{j}\right).\left(-u_2t\hat{i}-\frac{1}{2}gt^2\hat{j}\right)=0$$

$$\therefore\ t=\frac{2\sqrt{u_1u_2}}{g}$$

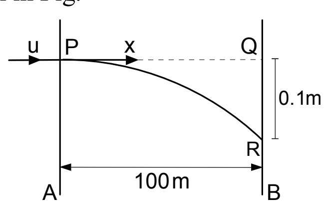
d) Separation between them when displacement is perpendicular to $X=(u_1+u_2)t=\frac{(u_1+u_2)2\sqrt{u_1u_2}}{g}$

Problem : 124

Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.

Sol. The situation is shown in Fig.

$$d=u\sqrt{\frac{2(h_1-h_2)}{g}}$$



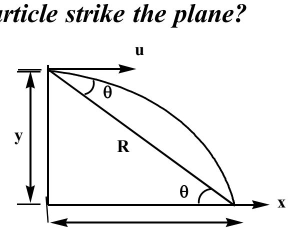
$$100=u\sqrt{\frac{2\times0.1}{9.8}}$$

$$u=700m/s$$

Problem : 125

A particle is projected horizontally with a speed "u" from the top of plane inclined at an angle "θ" with the horizontal. How far from the point of projection will the particle strike the plane?

Sol.



$R=\sqrt{x^2+y^2}$ $\left(\frac{y}{x}=\tan\theta\right)$

$=\sqrt{x^2+(x\tan\theta)^2}=x\sqrt{1+\tan^2\theta}$

$=x\sec\theta$

$x=ut;\ y=\frac{1}{2}gt^2;\quad \frac{y}{x}=\frac{1}{2}\frac{gt^2}{ut}$

$\tan\theta=\frac{gt}{2u}\ ;\ t=\frac{2u}{g}\tan\theta$

$x=ut=\frac{2u^2}{g}\tan\theta;\quad \therefore R=\frac{2u^2}{g}\tan\theta\sec\theta$

Problem : 126

An aeroplane is flying in a horizontal direction with a velocity of 600 km/hour at a height of 1960 m. When it is vertically above a point A on the ground, a body is dropped from it. The body strikes the ground at a point B. Calculate the distance AB.

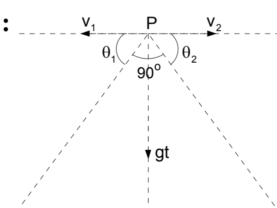
Hint : $t=\sqrt{\frac{2h}{g}}$, $R=ut$

Ans : *20s, 3.33 km*

Problem : 127

Two particles move in a uniform gravitational field with an acceleration "g". At the initial moment the particles were located at same point and moved with velocities $u_1=0.8\ ms^{-1}$ and $u_2=4.0\ ms^{-1}$ horizontally in opposite directions. Find the distance between the particles at the moment when their velocity vectors become mutually perpendicular. (g = 10 ms^{-2})

Hint :



$t=\sqrt{\frac{u_1u_2}{g}}$, $x=(u_1+u_2)t$

Ans : 0.48m

Problem : 128

A boy aims a gun at a bird from a point, at a horizontal distance of 100m. If the gun can impart a velocity of 500 m/sec to the bullet, at what height above the bird must he aim his gun in order to hit it?

(take g = 10 m/sec²)

Sol : x = vt or 100 = 500 × t

t = 0.2 sec.

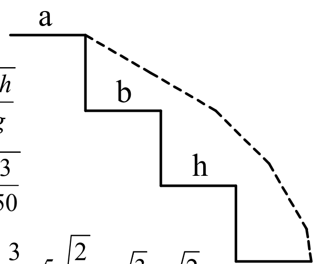
Now h = $0+\frac{1}{2}\times10\times(0.2)^2$

= 0.20 m = 20 cm.

Problem : 129

A staircase contains three steps each 10 cm high and 20 cm wide as shown in the figure. What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the edge of the lowest plane ? [g = 10 m/s²]

Sol : h = 30 cm



$S = 60cm = v\sqrt{\frac{2h}{g}}$

$= v\sqrt{\frac{2\times 30}{1000}} = v\sqrt{\frac{3}{50}}$

$\therefore v = \frac{60}{100}\sqrt{\frac{50}{3}} = \frac{3}{5}\times 5\sqrt{\frac{2}{3}} = \sqrt{3}\times\sqrt{2}$

=2.45 m/s

Problem : 130

An enemy plane is flying horizontally at an altitude of 2 km with a speed of 300 ms⁻¹ .An armyman with an anti - aircraft gun on the ground sights ht enemy plane when it is directly overhead and fires a shell with a muzzle speed of 600 ms⁻¹. At what angle with the vertical should the gun be fired so as to hit the plane ?

Ans : 30^0

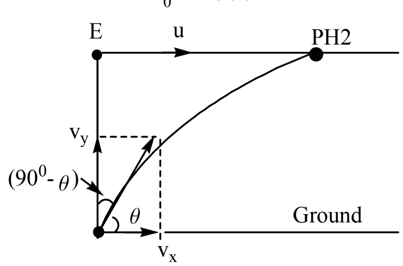
Sol. Let G be the position of the gun and E that of the enemy plane flying horizontally with speed

$u = 300\,ms^{-1}$, when the shell is fired with a spped v_0 is $v_x = v_0\cos\theta$

Let the shell hit the plane a point P and let t be the time taken for the shell to hit the plane. It is clear that the shell will hit the plane, if the horiazontal distance EP travelled by the plane in time t = the distance travelled by the shell in the horizotal direction in the same time, i.e.

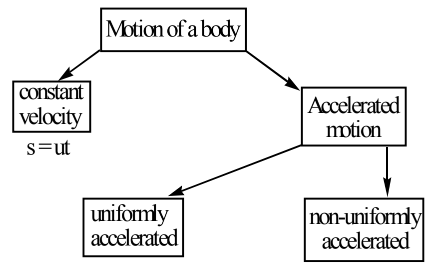
$u\times v = v_x \times t \; or \; u = v_x = v_0\cos\theta$

$or \;\; \cos\theta = \frac{u}{v_0} = \frac{300}{600}$



$= 0.5$ or $\theta = 60^0$.

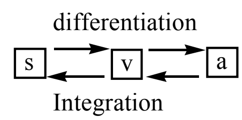
Therefore, angle with the vertical = $90^0 - \theta = 30^0$.



$v = u + at$

$v = \frac{ds}{dt}\,; a = \frac{dv}{dt} = v\frac{dv}{ds}$

$s = \left(\frac{u+v}{2}\right)t$



$v^2 - u^2 = 2as$

$\int ds = \int v dt$

$S = ut + \frac{1}{2}at^2$

$\int dv = \int a\, dt$

$S_n = u + a\left(n - \frac{1}{2}\right)$

$\int a ds = \int v dv$

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ALL THE BEST , BY

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